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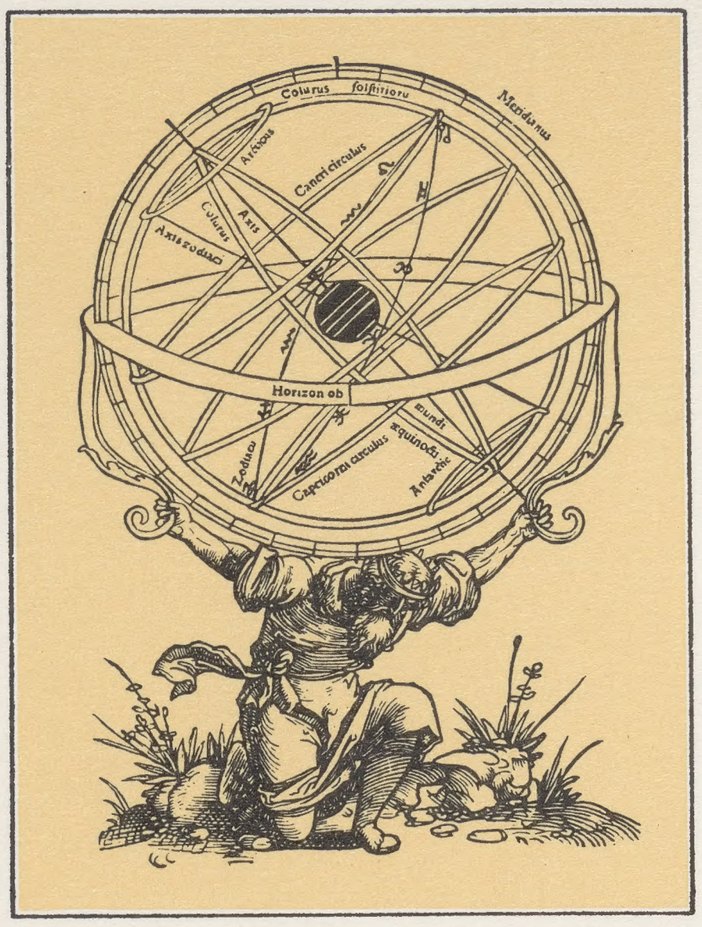

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An Important Mathematical Manuscript

23 SAUNDERSON or SANDERSON (Nicholas, *F.R.S.*). Of the Algorithm of Fluxions. Manuscript on 149 leaves. 4to, with 9 beautifully drawn folding plates of diagrams, contemporary (early 18th century) boards, calf back, lettered Sanderson's Fluxions. £15

An important manuscript of one of the most remarkable achievements in the history of mathematics. Sanderson lost not only his sight, but his eyes at the age of 12 months from small-pox. Shewing a remarkable aptitude for mathematics, he was given encouragement by parents and teachers, and eventually by Whiston, Lucasian Professor of Mathematics at Cambridge. In 1711 he was given a degree by royal decree, in order that he might succeed Whiston as Lucasian Professor. RICH - 56

The present MS. is an exposition of the calculus invented by Newton and Leibnitz, the most valuable technique for calculation that has yet been devised. Sanderson published nothing during his lifetime. *D.N.B.* states that some of his MSS. are extant in the University Library, Cambridge. The *Method of Fluxions* was published in 1751. He was made F.R.S. in 1719. Manuscript and endleaves are of the same paper, with a bold Pro Patrem watermark ; the folding plates are on special drawing paper. Though the practice of taking down lectures was more common in the 18th century than now, examples of copies of Sanderson's lectures have not survived. In any case, such lectures could have not have been of the complexity of the present manuscript. It must be emphasised that there was no printed version in Sanderson's lifetime, and that the present MS. is not only a scientific document of major importance but a remarkable example of the intellectual achievement of a totally blind man.

The watermarks are : (i) in the text. Heawood 3700 (England, Essex. 1747), and (ii) in the plates, Heawood 3148 (England, Essex, 1747), from which it is fair to assume that the MSS. was written about 1747, probably with the published book of 1751 in mind.

61

HERBERTUS PROFESSOR THEOLOGIÆ REGIUS CANTABRIGIÆ
NATUS OCTOB. 19. 1667. OBIIT FEB. 24. 1746 ÆTAT. SUÆ 79

Of the Algorithm of Fluxions

Definitions

Let ab represent any moment of time (whether finite or infinitely small it matters not) terminated by the 2 Instants a & b. Let x be y^e value of any flowing or growing quantity, at y^e Instant a, whose velocity at y^e Instant is such, that if it was to flow during the whole moment, a b, w^th y^e Velocity, it would gain a certain Increment, represented by (fluxion $\dot{x}$) then is y^e Quantity $\dot{x}$ call'd the fluxion of x at y^e Instant a, when the Value of the flowing quantity was x.

Scholium.

From y^e Definition it appears, y^t if x flow uniformly, its increm^t gain'd in y^e time a b, will be y^e same, as its fluxion above defined. But if x does not flow uniformly, i.e. if its velocity at y^e Instant b be not the same as its velocity at y^e Instant a, y^n its Increment in y^e time a b, is not y^e same, w^th its fluxion above defined, but will differ more or less from it, according as y^e time a b, is greater or less. But if y^e time (a b) be taken infinitely small y^n tho' y^e Velocity of x be not y^e same, mathematically speaking at y^e Instant, b, with y^e Vel. at y^e Inst. a, yet the Difference being infinitely small in respect of y^e whole velocity, it may safely be neglected, where y^e finite Ratios of y^e fluxions are only considered, and so y^e increment, & the

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fluxion above defined may be taken for one ano-
ther; i.e. the quantity x for so small a time
may be looked upon, as flowing uniformly—

Cor. 1. The fluxion of a constant quantity, is nothing.

Cor. 2. The fluxion of y^e whole, equalls y^e fluxions
of all its parts. Thus if r be a constant quantity
& x. y, & z, flowing ones, y^e fluxion of r + x + y − z will
be ẋ + ẏ − ż N.B. When different fluxions are
compared they are all supposed, to be generated uniform-
-ly, in y^e same time. Note also y^t a constant equality
of fluents, always implies an Equality of fluxions
but not vice versa, unless y^e fluents be generated
in equall times.

Lemma —

If 2 Quantities v & x. arithmetically expressed
be supposed to flow uniformly, y^e product of their
Multiplication, will not flow uniformly, but with
a Velocity equably accelerated; i.e. its continual
Increments, gain'd in successive equall times will
not be equall, but in Arithmetical Progression.

Dem: Let y^e time ad. be divided into equal parts
ab, bc. cd, & let u & x be the Values of y^e flow-
ing Quantities at y^e Instant a, then because these
quantities are supposed to flow uniformly, their
Values will be at y^e Instant. b. v + v̇ & x + ẋ, at
y^e Instant c v + 2v̇, & x + 2ẋ, at y^e Instant. d.
v + 3v̇ & x + 3ẋ, therefore their product will be at y^e
Instant a, vx, at y^e Inst. b, vx + vẋ + xv̇ + v̇ẋ, at
y^e Instant c, ux + 2uẋ + 2xu̇ + 4u̇ẋ at y^e Inst. d.
ux + 3uẋ + 3xu̇ + 9u̇ẋ. Subtract now y^e product
at y^e Inst. a, from y^e product at y^e Inst. b, and

* The Increm.t of y.e product at the Instant b, above w.t the product was at y.e Inst.t x, is $\dot{u}x + x\dot{u} + \dot{u}\dot{x}$, consisting of three parts. I say then, y.t if y.e moment of time, o b be taken infinitely small, y.t last of y.e three parts, will be infinitely less, y.n either of y.e other two.

For. 1.t let us compare $\dot{u}\dot{x}$ & $u\dot{x}$, and we shall have $\dot{u}\dot{x} : u\dot{x}$ as $\dot{u}$ to u but $\dot{u}$ is infinitely less y.n u, because generated in an infinitely small time $\frac{o}{Hyp.}$ &c. $\dot{u}\dot{x}$ is infinitely less y.n $u\dot{x}$ & by the like comparison it will also be found, $\dot{u}\dot{x}$ is infinitely less y.n $x\dot{u}$: therefore if $\dot{u}\dot{x}$ be taken away from the other two parts, the fluxion, of its product will be justly expressed, by $x\dot{u}$ & $u\dot{x}$. because $u\dot{x} + x\dot{u}$ is to $u\dot{x} + x\dot{u} + \dot{u}\dot{x}$ ultimately is a Ratio of Equality, i.e. it will terminate in that Ratio.

Prob. relating to ultimate Ratios —

Let x be a variable quantity, & let $4xx + 3x = a$, & $2xx + x = b$ then is plain, y.t when x is infinite. a & b, will both be infinite, when, $x = o$, a & b, will be nothing Quare w.t will be the ultimate Ratio of a to b whether, increasing or decreasing in Infinitum. a —

Before we give a direct Answer to y.e Question, it is to be observed that the Quantities a & b, while they have any being, will be in a less Ratio y.n that of 3 to 1, & in a greater y.n y.t of 2 : 1. For $4xx + 3x$ is to $2xx + x$ :: 3 : 1. as appears from the product of y.e Extreams & means. but $4xx + 3x$ is a less quantity y.n $4xx + 3x$. Ergo $4xx + 3x : 2xx + x$, in a less Ratio y.n y.t of 3 to 1. But $4xx + 3x = a$ & $2xx + x = b$, Ergo. a : b in a less Ratio y.n that of 3 to 1.

there will remain $uix + xi + iix$. for y^{e} Increm^{t}. gained by the product in y^{e} time ab. subtract again the product at y^{e} Instant b. from the product at y^{e} Instant c, & there will remain $uix + xi + 3iix$. for the Increm^{t}. gained in the time bc. Lastly. subtract the Product at y^{e} Inst^{t} c. from the product at y^{e} Inst^{t} d & there will remain $uix + xi + 5iix$. for y^{e} Increm^{t}. gain'd in y^{e} time cd. Therefore the continual Increm^{ts}. gain'd by y^{e} products in y^{e} succesive equal times ab, bc, cd, will not be equal but in Arithmetical Progression — —

N.B. If y^{e} times ab, bc, cd, be infinitely small, the Quantities iix. 3iix. 5iix. will be infinitely less y^{n} the other parts of y^{e} Increm^{ts}. as will easily appear, by comparing them *

Again $4xx+2x : 2xx+x :: 2 : 1$. But $4xx+3x$ is a greater Quantity y^n $4xx+2x$ ergo $4xx+3x : 2xx+x$ i.e. $a : b$ in a greater Ratio y^n 2 : 1. —

To come to the Point. Since $a = 4xx+3x$ & $b = 2xx+x$, $a : b :: 4xx+3x : 2xx+x$, divide the two last Quantities by x, & you'l have, $4x+3 = 2x+1$. Hence it is easy to see that the less the Quantities, a & b are, the nearer the Quantity x is to nothing, the nearer y^e Ratio of $4x+3$, to the Ratio of $2x+1$ will approach, to y^e Ratio of 3 : 1. When x is less y^n any assignable quantity, the Ratio of $4x+3$. to the Ratio of $2x+1$, or y^e Ratio of $a : b$, will approach nearer to y^e ratio of 3 to 1, y^n by any assignable difference, when x vanish=es, then they will terminate in the Ratio of 3 to 1. Ergo y^e ultimate Ratio of y^e evanescent quantities, a & b will be y^e 3 : 1. The Ratio of 3 to 1 continues to be a Ratio, after y^e Quantities a & b, are banished, but then it ceases to be the Ratio of a to b. Again since a is to b. as $4x+3$ is to $2x+1$, divide y^e last quantities again by x, & you'l have, $a : b :: 4+\frac{3}{x} : 2+\frac{1}{x}$. This is general. but it is easy to see, y^t y^e greater the Quantities all b are, 1, 2. the nearer the quantity x approaches to infi=nity, the less will be the Quantities $\frac{3}{x}$ & $\frac{1}{x}$ & the nearer will the Ratio of $4+\frac{3}{x}$ to $2+\frac{1}{x}$ approach, to y^e Ratio of 4 to 2 or 2 to 1. Let x be greater y^n any assignable quantity. & the Quantities $\frac{3}{x}$ & $\frac{1}{x}$ will be less, y^n any can be assigned, & the Ratio of $4+\frac{3}{x}$ to $2+\frac{1}{x}$ or of $a : b$ will approach nearer to y^e Ratio of 2 : 1. than by any assignable difference &c. when x is infinite & the quantities $\frac{3}{x}$ & $\frac{1}{x}$ entirely banish'd the Ratio of a to b. will terminate in the Ratio of 2 to 1.

Thus we have found 2 limits between wch ye Rate of a to b will always consist, to wch it may one way or other approach, beyond wch it cannot pass, & at wch it can never arrive 'till the Quantities whereof it is ye Rate, loose their being, either in nothing or infinity; as often as any limits of yt kind are finite Rates it must be the business of Mathematicks, to find them out —

Hence we have another Definition of a Fluxion If a & b, be any two variable magnitudes, whatever, finite or infinitely small, which are to one another, in their first Rate of Nascent Increments, or in the last Rate of their Evanescent Decrements —

Prob: 1st

To find the fluxion of y^e product arising from the continual Multiplication of any number of flowing Quantities whatsoever.—

Case 1st Let u & x be the values of two flowing Quantities, at any given Inst^t of time & let these Quantities, be supposed to flow uniformly, as in y^e last Lemma; then it's plain, that a moment before the given Instant y^e two Values were $u - \dot{u}$ & $x - \dot{x}$ their product, $ux - u\dot{x} - x\dot{u} + \dot{u}\dot{x}$ Therefore y^e Increment gained by the product in y^t moment, was, $u\dot{x} + x\dot{u} - \dot{x}\dot{u}$, but the Increment gained by the products in an equall moment immediately following y^e given Inst^t was found to be $u\dot{x} + x\dot{u} + \dot{u}\dot{x}$, & y^t latter Increment (by the above quoted Lemma) exceeds as much the true fluxion of y^e Product as y^e other wants of it. Therefore the true fluxion of y^e Product ux at the Instant of time y^t the factors are $\& x$ is $u\dot{x} + x\dot{u}$. Q & D.

Otherwise thus. The Increment gain'd by y^e Product in the time ab (see y^e Lemma) is $u\dot{x} + x\dot{u} + \dot{u}\dot{x}$, let y^e time ab, be infinitely small, y^t 'tis evident y^t the Quantity $\dot{u}\dot{x}$ being infinitely less y^n the rest may be neglected for it's insignificancy so that both y^e increment & fluxion of y^e product, will be $u\dot{x} + x\dot{u}$. Increase now the time from an infinitely small to a finite moment, $u\dot{x}$ & $x\dot{u}$ will increase with the same. as is evident from the Def: of a fluxion. Q & D $u\dot{x} + x\dot{u}$ will still be the fluxion of y^e product ux.

Case 2d Let there be now 3 flowing Quantities, u, x, & y, and the fluxion of y^d product, uxy, or of the product ux × y will be the fluxion of ux, multiplied into y, together with the fluxion of y multiplied into ux by by y^e last Case. But y^e fluxion of ux was there found to be $\dot{u}x + x\dot{u}$ (or $\dot{u}x + u\dot{x}$) & this multiplied into y, gives $uy\dot{x} + xy\dot{u}$. Therefore the whole fluxion of y^e product uxy is $uxy + uy\dot{x} + xy\dot{u}$

Case 3d In like manner if there be four flowing Quantities, the whole fluxion of y^e Product, uzyx, or y^e Product of uxy × z will be found to be $uxy\dot{z} + uz\dot{y} + uy\dot{z}x + xyz\dot{u}$. &c &c.—

Cor: If a be a standing Quantity & u a variable one, the fluxion of y^d product au, will be $a\dot{u}$.

Prob. 2d

To find the fluxion of any Power of any flowing Quantity, whether the Index of y^e Power be integral or fractional, affirmative or negative.

Case 1st Let u, x, y, z, in the last problem be all supposed equal to one another, then will $ux = x^2$ & its fluxion $\dot{u}x + x\dot{u}$ will be $2\dot{x} \times x$, therefore y^e fluxion of x^2 will be $2\dot{x} \times x$, or $2\dot{x} \times x^{2-1}$. Again $uxy = x^3$ & its fluxion $uxy + uy\dot{x} + xy\dot{u}$, will be $3\dot{x} \times x^2$ or $3\dot{x} \times x^{3-1}$ Again $uxyz = x^4$ therefore y^e fluxion of x^4, will be $4\dot{x} \times x^{4-1}$ & universally, the fluxion of x^m, will be $m\dot{x} \times x^{m-1}$; provided the Index m, be integral & affirmative.

Case 2d Let it now be required to find y^e fluxion of x^{-m} or $\frac{1}{x^m}$, in order to which make $\frac{1}{x^m} = z$ then $z \times x^m = 1$ & taking the fluxions on both sides, we have $\dot{z} \times x^m + mz\dot{x} \times x^{m-1} = 0$ (for the fluxion of 1 is 0)

Therefore $\dot{z} \times x^m = - m\dot{x} \times x^{m-1} = - \frac{m\dot{x} \times x^{m-1}}{x^m} =$ $\dot{z}$
$= - mx \times x^{-1}$ Therefore $\dot{z} = - m\dot{x} \times x^{-1} = - m\dot{x} \times x^{-m-1}$.
do y^e from y^e two last Cases it is now easy to conceive
y^t if the index m be a whole number whether
affirmative or negative the fluxion of $x^m = m\dot{x} \times$
x^{m-1} —
Case 3^d Let it now be required to find the fluxion
of $x^{\frac{m}{n}}$ —
To do this. Let $x^{\frac{m}{n}} = z$. then will $z^n = x^m$ Therefore
$nz\dot{z} \times z^{n-1} = m\dot{x} \times x^{m-1}$. from the 2 foregoing
Cases. Therefore dividing Equals by Equals.
* (viz. $nz\dot{z} \times z^{n-1}$ by z^n, & $m\dot{x} \times x^{m-1}$ by x^m but
$z^n = x^m$) we have $z\dot{z} \times z^{-1} = m\dot{x} \times x^{-1}$ & $\dot{z} \times z^{-1}$
$= \frac{m}{n}\dot{x} \times x^{-1}$ multiply both sides by z & we have
$\dot{z}$ or y^e fluxion sought $= \frac{m}{n}\dot{x} \times z \times x^{-1} = \frac{m}{n}\dot{x} \times$
$x^{\frac{m}{n}-1}$. so y^t at last it appears y^t whether the
index be affirmative or negative, integral or
fractional y^t the fluxion of x^m will be $m\dot{x} \times$
x^{m-1}.
Cor. 1^t The Fluxion of y^t Quantity $x^m y^n$ or $x^m \times y^n \times$
$\times z^{-p}$ by the 2 foregoing Problems will be $m\dot{x} \times$
$x^{m-1} \times y^n \times z^{-p} + n\dot{y} \times y^{n-1} \times x^m \times z^{-p} - p\dot{z} \times$
$\times z^{-p-1} \times x^m \times y^n$ —
Cor. 2^d The Fluxion of any Fraction $\frac{x}{y}$ will be
$\frac{\dot{x} \times y - \dot{y} \times x}{yy}$ as will appear either from the
foregoing Corollary, by considering, $\frac{x}{y}$ $x' \times y^{-1}$; or
rather thus; make $\frac{x}{y} = z$, then will $yz = x$ &
$y\dot{z} + z\dot{y} = \dot{x}$. Therefore $y\dot{z} = \dot{x} - z\dot{y} = \dot{x} - \frac{x\dot{y}}{y}$
$= \frac{y\dot{x} - x\dot{y}}{y}$, therefore $\dot{z}$ or the fluxion sought $=$
$= \frac{y\dot{x} - x\dot{y}}{yy}$. — — —

Explication of Lemma 2d Lib. 2d Newt. Princip:

"Momentum genita &c--

The coefficient of any Quantity as a, in another as b, is yt Quantity which expresses, how often a, is contained in b., & which arises from dividing (by), by, a, thus yt coefficient of a in ab, is b. of a in b is $\frac{b}{a}$ of a^m is a^{m-1} of a^{-n} is a^{-n-1}, of a in a^n b^m is a^{n-1} b^m of b, in a or a; is 1. or 1. This understood let it be figured from the above The. to find the Fluxion of $x^m y^n z^{-p} = \frac{x^m y^n}{z^p}$

Find the Fluxion of x is $\dot{x}$, the index of its power m, & the Coefficient of x is $x^{m-1} y^n z^{-p}$ these continually multiplyd together, make $m\dot{x}x^{m-1}y^n z^{-p}$

Again, the fluxion of y is $\dot{y}$, its index, n, & its coefficient $y^{n-1} x^m z^{-p}$ Their product $n\dot{y}y^{n-1}x^m z^{-p}$.

Lastly yd fluxion of z^{-p} is $\dot{z}$ its index $-p$ & its coefficient is $z^{-p-1}x^m y^n$ & the product of yd multiplication of these is $p\dot{z}z^{-p-1}x^m y^n$ - wherefore the fluxion of yd product wrought out is $m\dot{x}x^{m-1}y^n z^{-p} + n\dot{y}y^{n-1}x^m z^{-p} - p\dot{z}z^{-p-1}x^m y^n$

Before we leave yt subject, another thing more is to be added, wch is to find yd fluxion of any power of a flowing Quantity, in a manner different from yt we have hitherto made use of.

We have already taken notice yt yd fluxion or Velocity of flowing Quantities, are in the first Ratio of their Nascent Increments or in yd last Ratio of their Evanescent Decrements & this Idea of Fluxions alone suggests to a very easy method of finding them. For thus,

Let x be any flowing Quantity, whether uniform or not it matters not, & let it be required to find the fluxion of xx, i.e. let it be required to compare the fluxion of xx with y^e fluxion of x, at their first setting out. Let x in any time whether finite or infinitely small it matter not become x + a, & xx in y^e same time will become xx + 2ax + aa, so y^t their Contemporary increments will be as a to 2ax + aa, or dividing by a, as 1 to 2x + a. this will be universally, y^e Ratio of y^e contemporary increments of x & of xx. but y^e Vel. of x & xx will not be in y^e Ratio (i.e. their first Velocities) but rather in y^e 1st Ratio, from w^ch this Ratio took its Rise. Now it is manifest y^t the less y^e Increment a is, the nearer will y^e Ratio of 1 to 2x + a, approach to y^e Ratio of 1 to 2x. when a is infinitely small y^e Ratio of 1 to 2x + a, will approach infinitely near to y^e Ratio of 1 to 2x. Therefore y^e Ratio of 1 to 2x was y^e first Ratio of y^e nascent Increments of x & xx. Therefore y^e fluxion of xx is to y^e fluxion or Vel. of x as 1 to 2x, or as $\dot{x} : 2x\dot{x}$. Therefore if y^e former represent y^e fluxion of x, the latter will represent the fluxion of xx. Again, while x becomes x + a, x^3 will become $x^3 + 3axx + 3aax + aaa$. And their Contemporaneous Increments will be as a, to $3ax^2 + 3a^2x + a^3$ or as 1, to $3xx + 3ax + a^2$. but it is plain y^t this Ratio, took its Rise from the Ratio of 1 to 3xx, therefore the fluxion of $x : x^3 :: 1 : 3xx$ or $\dot{x} : 3xx\dot{x}$ & so of all Integ^l all powers, as to the negative & fractional ones they may be deduced from y^e Integral ones, as before.

Examples of finding the fluxions of fluents—

1— The Fluxion of xx is $2\dot{x} \times x$.

2— Of x^5 $5\dot{x} \times x^4$

3— Of $\frac{1}{x} = x^{-1} = -\dot{x} \times x^{-2} = \frac{-\dot{x}}{xx}$

4— Of $\frac{1}{x^3} = x^{-3} = -3\dot{x} \times x^{-4} = \frac{-3\dot{x}}{x^4}$

5— Of $\frac{a}{x^5}$ or $a \times x^{-5}$ / a being a fix'd Quantity / $\frac{5a\dot{x}}{x^6}$

6— Of $\sqrt{x}$ or $x^{\frac{1}{2}} = \frac{1}{2}\dot{x} \times x^{-\frac{1}{2}} = \frac{\frac{1}{2}\dot{x}}{\sqrt{x}}$

7— Of $\sqrt[3]{x}$ or $x^{\frac{1}{3}} = \frac{1}{3}\dot{x} \times x^{-\frac{2}{3}} = \frac{\frac{1}{3}\dot{x}}{\sqrt[3]{xx}}$

8— Of $x\sqrt{x}$ or $x' \times x^{\frac{1}{2}}$ or $x^{\frac{3}{2}} = \frac{3}{2}\dot{x} \times x^{\frac{1}{2}} = \frac{3}{2}\dot{x} \times \sqrt{x}$

9— Of $\frac{1}{\sqrt{x}}$ or $1 \times x^{-\frac{1}{2}} = \frac{1}{2}\dot{x} \times x^{-\frac{3}{2}} = \frac{\frac{1}{2}\dot{x}}{\sqrt{x^3}}$

10— Of $\frac{x}{\sqrt[3]{xx}}$ or $x' \times x^{-\frac{2}{3}}$ or $x^{\frac{1}{3}} = \frac{1}{3}\dot{x} \times x^{-\frac{2}{3}} = \frac{\frac{1}{3}\dot{x}}{\sqrt[3]{xx}}$

11— Of $\frac{aa}{a+x}$ by 4° last Coral. will be $\frac{-aa\dot{x}}{aa+2ax+xx}$. or it may be proved universally thus. by making $\frac{aa}{a+x} = y$, for then we shall have $aa = ay + xy$. but y° fluxion of ay is $a\dot{y}$ & the fluxion of xy is $x\dot{y} + y\dot{x}$. therefore the fluxion of $ay + xy$ is $a\dot{y} + x\dot{y} + y\dot{x}$ or $\overline{a+x} \times \dot{y} + y\dot{x}$ but the fluxion of aa is nothing, because a is a constant Quantity, therefore $\overline{a+x} \times \dot{y} + y\dot{x} = 0$. $a+x \times \dot{y} = -y\dot{x} = \frac{-aa\dot{x}}{a+x}$. therefore $\dot{y} = \frac{-aa\dot{x}}{a+x}$ as above. — — —

12. When r is a constant Quantity the fluxion of $\sqrt{rr-xx}$, will be found by putting $y = \sqrt{rr-xx}$. for y° $yy = rr - xx$ & $2y\dot{y} = -2\dot{x}x$ & $y\dot{y} = -x\dot{x}$ therefore $\dot{y} = \frac{-x\dot{x}}{y} = \frac{-x\dot{x}}{\sqrt{rr-xx}}$

13. Of $\frac{x}{a}$ will be $\frac{a\dot{x}}{aa} = \frac{\dot{x}}{a}$

14. Of $\frac{x}{a+x} = \frac{a\dot{x}+x\dot{x}-x\dot{x}}{aa+2ax+xx} = \frac{a\dot{x}}{a+x^2}$

15 Of $\frac{a}{x} = \frac{-a\dot{x}}{xx}$.

16. Of $\sqrt{2rx-xx}$ or $\overline{2rx-xx}^{\frac{1}{2}} = \frac{r\dot{x}-x\dot{x}}{\sqrt{2rx-xx}}$ for. supposing $\sqrt{2rx-xx} = z$ we shall have $2rx-xx = zz$ & $r\dot{x}-x\dot{x} = z\dot{z}$ & conf. $\frac{r\dot{x}-x\dot{x}}{z} = \frac{r\dot{x}-x\dot{x}}{\sqrt{2rx-xx}} = \dot{z}$ 4° fluxion sought.

17 Of $\overline{ay-xx}^{\frac{3}{2}}$. Let $\overline{ay-xx}^{3} = z$ then $ay-xx = z^{\frac{1}{3}}$ & $a\dot{y}-2x\dot{x} = \frac{1}{3}z^{-\frac{2}{3}}\dot{z}$; $3az^{\frac{2}{3}}\dot{y} - 6z^{\frac{2}{3}}x\dot{x} = \dot{z}$ (by substituting $ay-xx^{4}$ for $z^{\frac{2}{3}}$) $3a^{3}y\dot{y} - 6axx\dot{y} + 3ax^{4}\dot{y} - a^{2}y^{2}\dot{x} + 12ayx^{3}\dot{x} - 6x^{5}\dot{x} = 4^{\text{o}}$ fluxion of $\overline{ay-xx}^{\frac{3}{2}}$ sought.

18 Of $\sqrt{xy+yy} = \frac{y\dot{x}+x\dot{y}+2y\dot{y}}{2\sqrt{xy+yy}}$.

19 Of $\sqrt{x^{5}+axyy} = \frac{ayy\dot{x}+2axy\dot{y}}{2\sqrt{x^{5}+axyy}}$

20 Of $\sqrt{ax+x+\sqrt{a^{4}+axyy}}$ may be found, by making $z = \sqrt{ax+xx+\sqrt{a^{4}+axyy}}$. for zz will equal the quantity $ax+xx+\sqrt{a^{4}+axyy}$ & $zz-ax-xx = \sqrt{a^{4}+axyy}$ therefore $2z\dot{z}-a\dot{x}-2x\dot{x} = \frac{ayy\dot{x}+2axy\dot{y}}{2\sqrt{a^{4}+axyy}}$ & z^{3} or 4° fluxion sought $= \frac{a\dot{x}+2ax\dot{x}+ayy\dot{x}+2axy\dot{y}}{2\sqrt{ax+xx+\sqrt{a^{4}+axyy}}\times 2\sqrt{a^{4}+axyy}}$

21. Of $\frac{3\sqrt{ax+xx}}{\sqrt{xy+yy}}$ will be found to be $\frac{a\dot{x}+x\dot{x}}{\sqrt[3]{ax+xx}}\times \sqrt{xy+yy}\ \frac{-y\dot{x}+x\dot{y}-2y\dot{y}}{2\sqrt{xy+yy}}\times\sqrt[3]{ax+xx}$

22 Of $\frac{bxx+cax+daa}{+dax}\times\sqrt{xx+aa}$. make $z = bxx+cax$ & $p = bxx+cxx+daa$, & $q = \sqrt{xx+aa}$ & then $pq = z$ Conf: $p\dot{q}+q\dot{p} = \dot{z}$. but $\dot{q} = \frac{x\dot{x}}{\sqrt{xx+aa}}$ & $\dot{p} = 2bx\dot{x}+ca\dot{x}$ wherefore if in 1st Equation $p\dot{q}+q\dot{p} = \dot{z}$ we restore their respective values we shall have $\frac{bx^{3}+caxx+daax}{\sqrt{xx+aa}}\times\dot{x}+2bx\sqrt{xx+aa}\times\dot{x}+ca\sqrt{x^{2}+aa}\times\dot{x} = \frac{3bx^{3}+2acxx+daax+2baax+ca^{3}}{\sqrt{xx+aa}} = \dot{z}$.

* The Ratio of Mm to hm is always equall to y^e Ratio of MD to TD ~~therefore~~ y^e ultimate Ratio of MD to TD is that of y^e ordinate to the subtangent, which will therefore be equal to $\frac{yx}{y}$, since the ult. Ratio of Mh to Mm, is y^t. of y to x and therefore $y : x :: y : \frac{yx}{y}$ =

Prob. 3 – Fig: 5th Tab. 1. Page.

It is required to draw a tangent to any given Point P as M of any given curve, as AM. whose Axis is AB.

Solution.

Draw two parallel Ordinates MP. mp. draw also the Chord MK. & produce it till it meets the Axis (produced if need be) in T. & compleat the Parallelogram PpmK. then will the Triangles PMm, be similar to y^e triangle PMT, & we shall have KM. Km, or Pp:: MP. PT. whence $PT = \frac{MP \times Pp}{KM}$. but now the Ordinate may be conceived to move away parallel to it self, till at last it arrives at MP, or at least comes infinitely near it, & during this whole motion, let y^e Line MT, be supposed so to turn on y^e Point M, as always to pass through m, then 'tis plain at y^e line MT will at last become a Tangent to y^e Curve in y^e Point M. & the Line PT a subtang^t. & if we call AP. x, PM y, we shall ultimately have Pp = ẋ PM = ẏ & y^e subtangent PT = $\frac{y\dot{x}}{\dot{y}}$ Compute therefore from the nature of y^e Curve the Quantity $\frac{y\dot{x}}{\dot{y}}$ & you will have the subtangent PT as in y^e following Examples.

Ex: 1–

Supposing all things as above, let AM be the common Parabolla whose Parameter let be p then we shall have, $px = yy$ + $p\dot{x} = 2y\dot{y}$ & $\dot{x} = \frac{2y\dot{y}}{p}$ & $y\dot{x} = \frac{2yy\dot{y}}{p}$ & $\frac{y\dot{x}}{\dot{y}} = \frac{2yy}{p}$ or y^e subtangent PT $= \frac{2yy}{p} = 2y \cdot \frac{2y}{p} = 2x = 2$ AP as in y^e Conic Sections.

If $n=m$, i.e. if $\frac{n}{m}=1$, the curve will become a right line, & coincide with y^e tangent, y^e subtang^t = the Abscis. If n be less than m, the subtang^t will be less than the Abscis, & y^e Curve convex towards y^e Axis. If n exceeds m, or $\frac{n}{m}$ exceeds Unity, the subtangent will be greater y^n the Abscis & the Curve Concave towards the Axis.

Example 2d

Let the nature of y^e curve be expressed by this Equation, $px^m = y^n$, then we shall have $pm\dot{x} \times x^{m-1} = n\dot{y} \times y^{n-1}$ & $m\dot{x} \times x^{\overline{m-1}} = n\dot{y} \times y^{\overline{n-1}}$ or $\frac{m\dot{x}}{x} = \frac{n\dot{y}}{y}$ hence $m\dot{x} = \frac{nx\dot{y}}{y}$ & $my\dot{x} = nx\dot{y}$ & $\frac{my\dot{x}}{\dot{y}} = nx$ & $\frac{y\dot{x}}{\dot{y}} = DT = \frac{n}{m} \times AP$. If $px = y$, then will $DT = AP$ & ∴ will be a right Line because $x : y :: 1 : p$. ##

Call the Power by y^e Hyperbola, p, and d, (by L'Hosp. Can.* Art. 101) $\frac{p}{x} = y$, but $\frac{p}{x} = p \times \frac{1}{x} = p \times x^{-1}$ therefore $p \times x^{-1} = y$, then will $DT = -2x$. the sign (−) signifying that y^e Subtangent does not lye, on y^e same side y^e Ordinate DM, as y^e Abscis AP. This is agreable with Con. Sect. Art. 107, where Conj. answers to our A, H to our P, D to our T. if $\frac{p}{x^3} = y^{-4}$ then $p \times x^{-3} = y^{-4}$ & $DT = -\frac{4}{3}$ of x.

Exam. 3d

Fig 6. Let AMBa, be an Ellipse Call AC t. CB. c, CP. x & DM. y. then we shall have from the nature of the Ellipse $cc - \frac{ccxx}{tt} = yy$, & Conj. $-\frac{2ccx\dot{x}}{tt} = 2y\dot{y}$. Therefore $\dot{x} = \frac{-tt}{ccx} \times y\dot{y}$ and $y\dot{x} = -\frac{tt}{ccx} \times yy\dot{y}$ & $\frac{y\dot{x}}{\dot{y}} = \frac{-tt}{ccx} \times y\dot{y}$ but y^e sign (−) signifying no more, than that y^e Tangent DT is not now to be on y^e same side of y^e Ord. with DC or x but on the contrary side—

Further, since $yy = cc - \frac{ccxx}{tt} = \frac{cctt - ccxx}{tt}$ $= \frac{cc}{tt} \times tt - xx$, if instead of yy in the former the Values of DT, we substitute its equal $\frac{cc}{tt} \times tt - xx$, we shall have now,

$TP = \frac{tt}{rcx} \times \frac{rc}{tt} \times tt - xx$. And if to this Value of TP, we add CD or x. we shall have $CT = \frac{tt}{x}$. Whence we shall have the following proportion for finding the Point T. $(v\&z)$ $CD : CA :: CA : CT$.

Prob: 4th

To determine, when a flowing quantity, becomes a Maximum or a Minimum. —

Solution —

The Fluxion of every flowing Quantity, is affirmative or negative, according as y^t Quantity increases or decreases, it follows, therefore, y^t when a Quantity becomes a Maximum or a Minimum its fluxion is equal to nothing. Take therefore the fluxion of such a Quantity in general, & then suppose it equal to nothing, & y^e Equation will determine the case wherein it is a maximum or a minimum. — —

Example 1st —

Let it be required to divide a given Quantity, as a, into 2 such parts that the product of their Multiplication may be y^e the greatest possible —

Here putting x & a − x for the two parts, their product of their Multiplication will be ax − xx whose fluxion is $a\dot{x} - 2x\dot{x}$ & Hyp: = 0 hence $x = \frac{1}{2}a$ which shows, y^t y^e product of y^e two pts will be the greatest possible when they are equal to each other.

Ex: 2d

Let it be required to divide a given number as a into 2 such parts, y^t one p^t multiplied into the square of y^e other may be the greatest possible.

Here put x for one part & a − x for its square, and [illegible] a − x for the other part, then will

the product be $-axx+x^3$ whose fluxion is $2axi$ $-3xxxi$ ex Hyp^o $=0$ hence $x=0$ or $\frac{2}{3}a$ which shews y^t if the part to be squared be $\frac{2}{3}$ of y^e whole & Cons^t the other part $\frac{1}{3}$. the product will be y^e greatest possible; & since in y^e foregoing Equation x was also found $=0$, it shews y^t if x be less than $\frac{2}{3}$ of a the product will be the least when when x or the p^t squared $=0$ —

Exam: 3^d —

Tis required to determine when the following Quantity becomes a maximum or a minimum (viz) x^3-18x^2+96x Hence $3xxxi-36xxi+96xi=0$ whence $x=4$ or 8. Therefore the Quantity proposed is a maxim: in one case & a minim: in the other. But to determine distinctly to which Case the maxim: belongs it must be observed that y^e fluxion of y^e Quantity proposed is either affirmative or negative according as y^e Quantity $xx-12x+32$ is so. But this Quantity when $x=0$ is affirmative Therefore the Quantity proposed is upon is upon the Increase till $x=4$ when it becomes a maxim: after which its diminish'd till $x=8$ in w^{ch} Case it becomes a Minim; then it increases again in infinitum.

Ex: 4:

Fig: 7 To find the greatest Random of a Ball, shot with a given Velocity.

Let AB be the Direction of y^e Ball, & by the force given it at A, be such as alone,

would have carried it from A to B, whilst y^e
Ball by his gravity alone w^d have described
tho' a known Space as A.S. then with AB &
be constant quantities; because y^e Velocity
of y^e Ball at A and y^e force of gravity are
supposed y^e same, whatever be y^e Inclination
ABC. Let the Horizontal Line AD be y^e
Diameter of y^e Ball & Draw DC & BC.
perpendicular to AD. Lastly compleat y^e
Parallelogram ABCD. This done 'tis evident
that whilst the Ball describes the Curve line
AB it would by its projectile force alone
have carried from A to C & by its gravity
alone from A to D. for y^e force of gravity
acting in a line parallel to DB will neither
help nor hinder the motion of y^e Ball to-
-wards the Line CB & by y^e same may be said
with respect to y^e projectile force & the line
DA. whence it follows, y^t while y^e ball,
by its projectile force alone w^d have passed
thro the Space AC by its gravity alone
it would have passed through AD or
BC since the Spaces through which heavy
Bodies fall from the rest are as y^e Squares
of y^e times, it follows, y^t as the Square of
y^e time wherein y^e Ball by its projectile
force alone w^d have described the Space AB
so is the Square of the time wherein the same
would have described the Space AC, i.e. (as
$AB^2 : AC^2$ or as $AK^2 : AD^2$) so is AD

Therefore $CD = s \times \frac{AD^2}{Ab^2}$ & $AD = \frac{Ab \times bB}{s}$.

Therefore the Random AD will be the greatest when $\underline{Ab \times bB}$ becomes a maximum, i.e. when $Ab \times bB^2$ becomes a maximum. Call AB, x. Bb, x, Ab, y, & in the present case we shall have $\dot{x} \times y + \dot{y} \times x = 0$. But $xx + yy = rr$ a constant Quantity, therefore $2x\dot{x} + 2y\dot{y} = 0$ hence $\dot{y} = -\frac{x\dot{x}}{y}$ & $\dot{y} \times x = -\frac{xx\dot{x}}{y}$ & $\dot{x} \times y + \dot{y} \times x = \dot{x} \times y - \frac{xx\dot{x}}{y} = \frac{yy\dot{x} - xx\dot{x}}{y} = \frac{rr\dot{x} - 2xx\dot{x}}{y} = 0$

Therefore $rr - 2xx = 0$ & $xx = \frac{1}{2}rr$, & $yy = \frac{1}{2}rr$.

Hence to give the Ball the greatest Random possible, the piece must be elevated to an Angle whose Sine & Cosine are equal, i.e. to an Angle of 45° Q.E.I.

Scholium

If the Piece be elevated to an Angle above 45° the Random of the Ball will be y^e^ same as if it was elevated to an Angle as much less than 45° because the Quantity $\frac{Ab \times bB}{s}$ will be y^e^ same in both Cases.

Example 5th

To find the most commodious Situation of a Plane, to be turn'd about an Axis by a wind whose course is in y^e Direction of y^e Axis.

Tab. 2. Fig. 1. Page.

Let both the Plane of y^e Page, & y^e Axis of Motion be supposed at some distance under it & imagin'd in an Horizontal Position. & let y^e Line A B be parallel to y^e Axis & let C D be y^e Plane in Question, in a Position perpendicular to y^e Horizon, resisting with its lower Edge upon y^e Axis of its Motion, & represented by its intersection, with the Plane of y^e Paper, in y^e Line C D. Let A C be perpendicular to C D. so that y^e Angle A B C may be the Inclination of y^e Plane to its Axis. & let C E be perpendicular & E G parallel to the Line A B. & let D G be perpendicular to y^e Line E G. Let A B^3 or y^e Cube of A B, represent the absolute force of y^e wind upon the Plane C D when directly exposed to its Current, then will the force of y^e same wind upon the Oblique Plane C D be diminish'd in the Ratio of A B to A C or of A B^3 to A C × A B^2. Say as y^e same wind, for y^e number of Particles acting now upon y^e oblique Plane C D, will by its Obliquity be diminish'd in y^e Prop.^n of C D to D G, or of A B to A C or of A C × A B^2 to A C^2 × A B. By this force the Plane C D if left to it self would move not directly before the wind as when directly exposed to its Current but in y^e Direction of the Line A E. This I say,

will be y^e Case of the Plane was absolutely at
Liberty, but in the present Case the Plane is
restrain'd from being moved in other Direction but
that of y^e Perpendicular F E to A B. by which
motion it is caused round its Axis & the Conatus
of y^e Plane to move in y^e Direction of S will
be to its conatus to move in y^e Direction F E. as S
S F or as A B to B C or as A B^2 × A B to A B^2 × S B.
therefore the Conatus of y^e Plane to move round
its Axis, must be represented by A B^2 × S B. Cwill
be a maximum when supposing A B Constant.
the Quantity A B^2 × S B is so. Call A B x & S B z &
we shall have, A B^2 S B × = xxz − xxx whose fluxion is xxż
− 3xxẋ in this Case = 0 whence xż − 3xẋ = 0
& ẋ = ż/3 Therefore the Effect of y^e wind will
be greatest when the Plane is inclined to y^e Axis
at an Angle whose Cosine is to y^e Radius as 1 : √3.
i. e. at an Angle of 54°. 44'. This is the Case
when the Plane is in a situation perpendicular
to y^e Horizon, & it will be y^e same in all
others. provided y^e Angle w^ch y^e Plane makes
with its Axis be y^e same. E D S

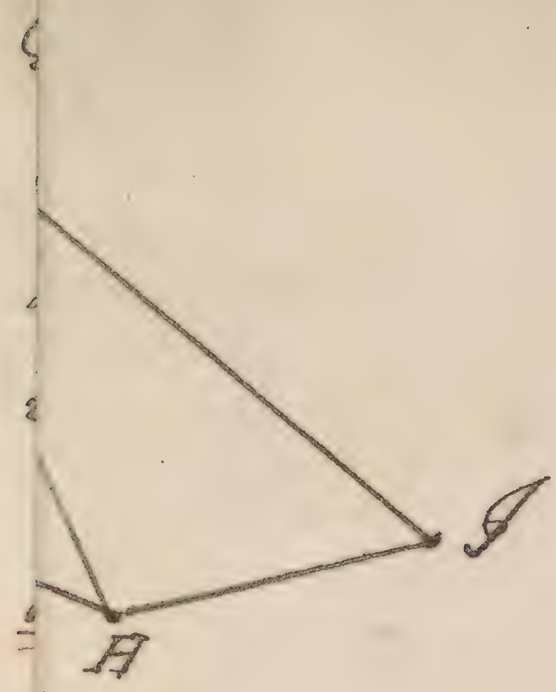
Example 4th

Fig 2.

To construct the frustrum of a Cone of a
given Base & Altitude w^ch moving in the
Direction of the Axis, with its smaller End against
the Parts of an uniform fluid, shall suffer
the least resistance possibly from it.

Let the Isosceles Triangle A B C represent a
Cone generated by its revolution round its Axis

A D &c. & let F D y^e be parallel to y^e Base & let
F H & y^e H. be perpendicular to the sides A B
& A C respectively & let F I parallel to A C cut
the Base B C in I. This supposed let us first
Enquire what effect a Stroke of particles moving in
y^e Direction A C would have upon y^e Cone when at
rest: in order to which. Let the Point F be struck
with one of these particles, & let A K represent
the absolute force with which the particle moves
& with which it would strike the Base of y^e
Cone if directly opposed to its Motion. Then
'tis plain, that the Point F receiving y^e Stroke
obliquely will only sustain the force of a
Stroke equale to F K. Let this force F K be
resolved into 2 forces F D & D K. & the former
of these F D being sustained by another, con-
trary Stroke & D K coming from G will have
no effect upon y^e Cone But y^e other force.
D K meeting with no opposition will have y^e
full effect upon y^e Cone to move it in
the Direction D K. if not otherwise reg-
-lain'd. Therefore the effect of a Particle at
the Point F in the Cone Surface, will be to
the effect of y^e same Particle at y^e Point
I in the Base as D K. to A K. or as A K × D
is to A K.^2 or (because of y^e similar Triangles
K F D. K A F. where K D. K F :: K F. K A as
K F. A K.^2 or as B C.^2 to A B.^2 but what hath
been said of y^e Points F & G will be equally
true of any other two correspondent Points
whatsoever. Therefore the Effect of any



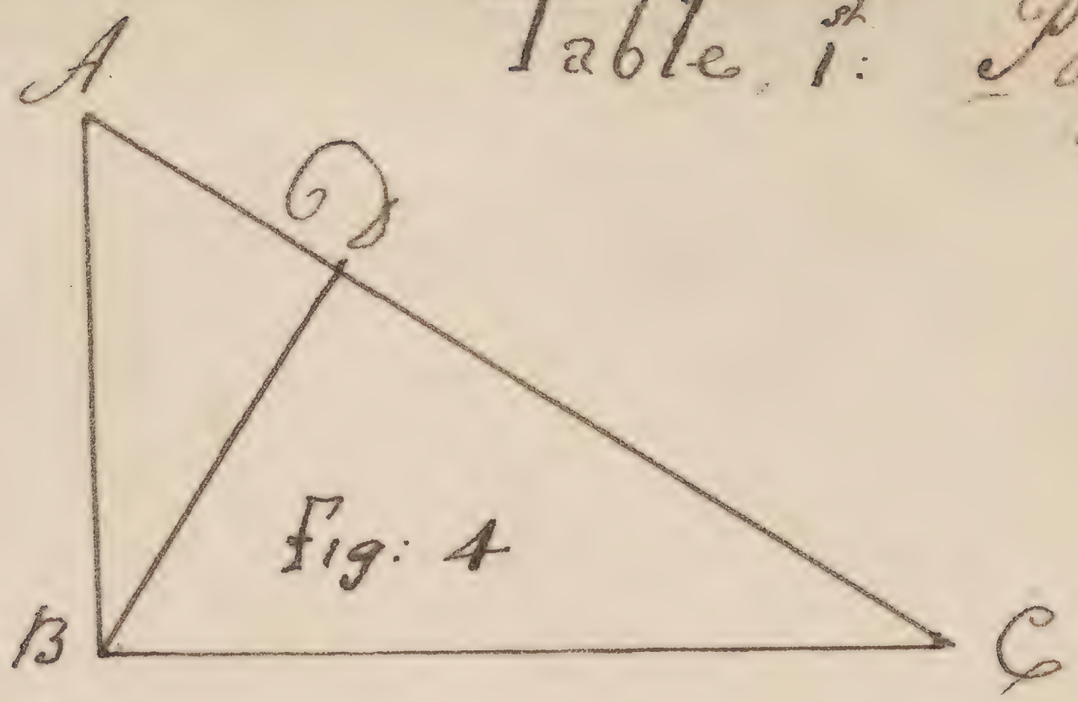


Fig: 4

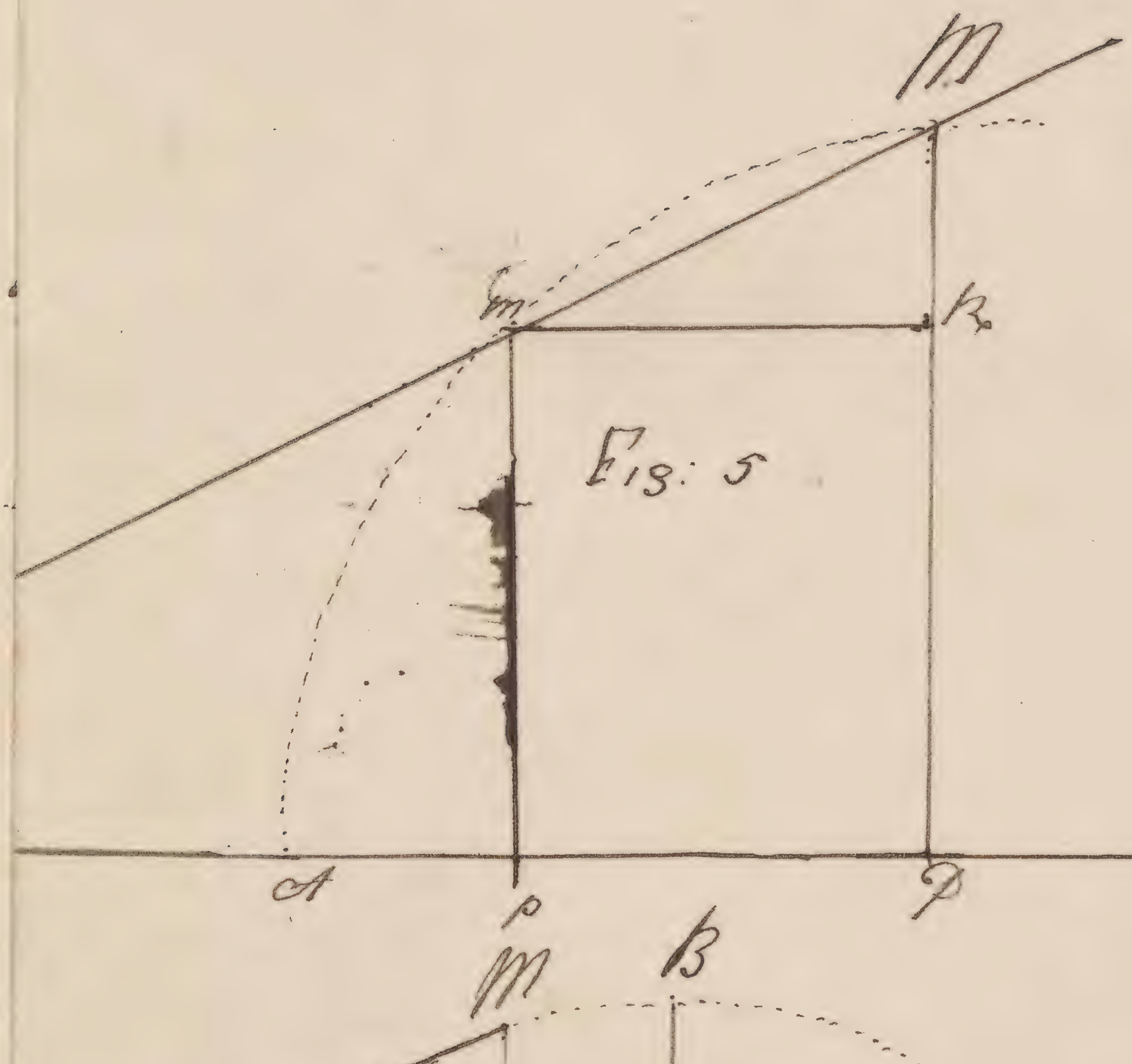


Fig: 5

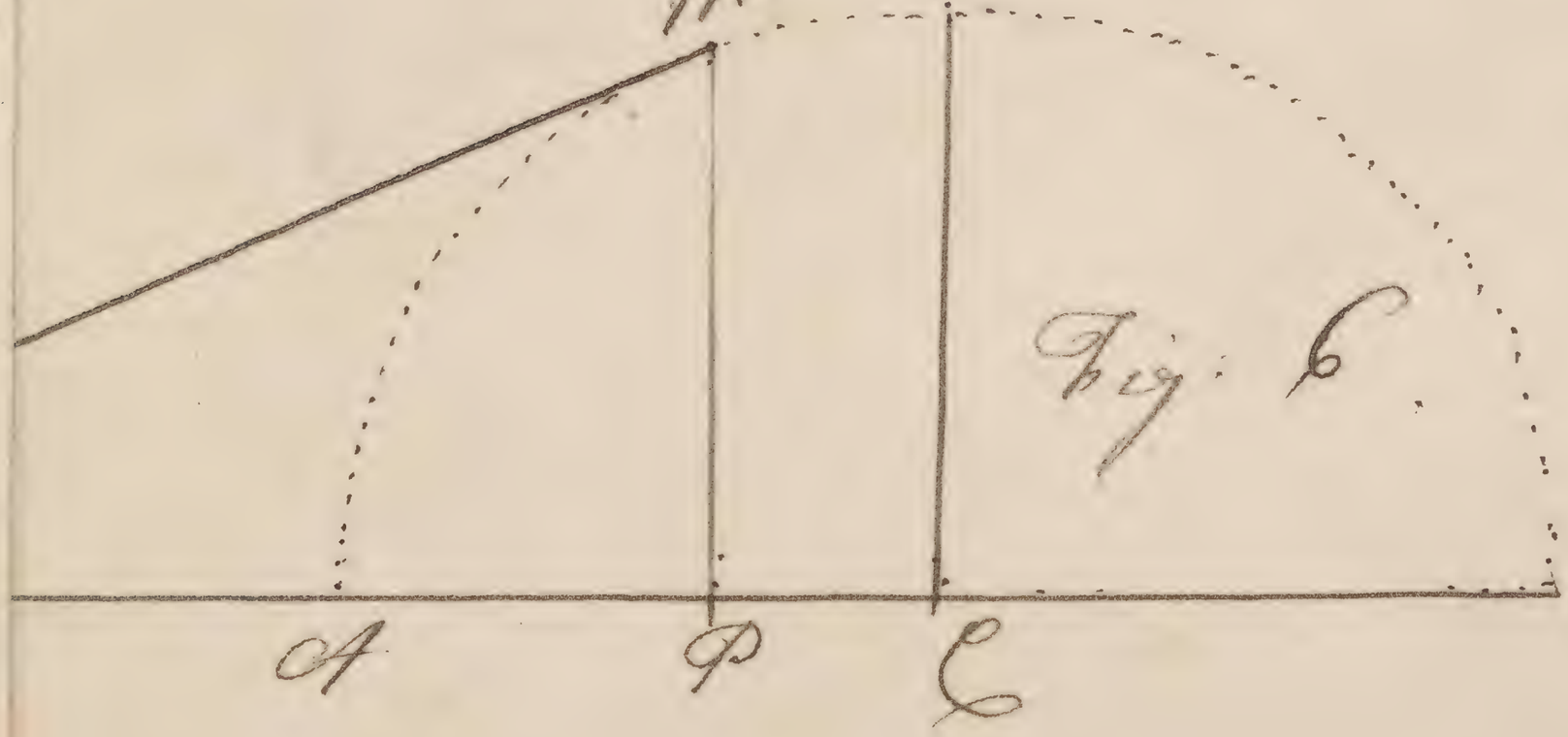


Fig: 6

Number of particles upon the Base of y^e Cone. will be to the effect of y^e same particles upon its surface as AB^2. BC^2 & since Action & reaction are equal, if we now suppose y^e parts of the fluid at rest & the Cone to move uniformly in it according to the Direction of its Axis. the Resistance the Base of y^e Cone would meet with (when moving with y^e same velocity) in y^e Direction AB, will be to y^e Resistance its surface would meet with moving in the Direction CA as AB^2. BC^2 Let then AB^2 represent y^e Resistance of y^e Base, & Comp. BC^2 y^e surface, & let the Cone FAG be cut of. Then will the Resistance of y^e Base BC in one Direction be to the Resistance FG in ~~y^e~~ other as y^e Area's of their Bases, y^t is, as the Squares of their Diameters. i.e. as BC^2. FG^2 or AB^2. AF^2. Therefore since AB^2 represents the Resistance of y^e Base BC — AF^2 will represent the Resistance of y^e FG. Therefore BF^2 will represent y^e resistance of the surface. From BC^2 the Resistance of y^e surface BAC subtract the Resistance of y^e surface FAG, and add AF^2 the Resistance of y^e Base FG, and we shall have the Resistance of y^e Frustum BFGC (when moving in the Direction ED) $= BC^2 + AF^2 - FD^2 = BC^2 + AD^2$ But $AD^2 = AE^2 - ED^2 = AE^2 - 2AE \times ED + ED^2$. Therefore the Resistance of the Frustum will be $BC^2 + AE^2 - 2AE \times ED + ED^2 = AB^2 - 2AE \times ED + ED^2$. This is upon a supposition

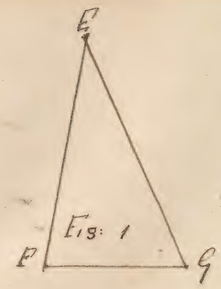


Fig: 1

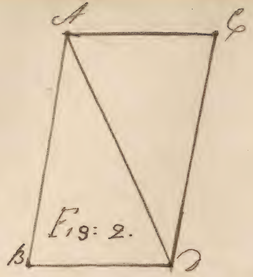


Fig: 2.

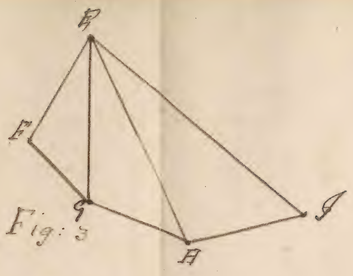


Fig: 3

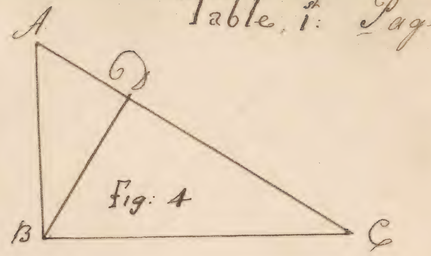


Fig: 4

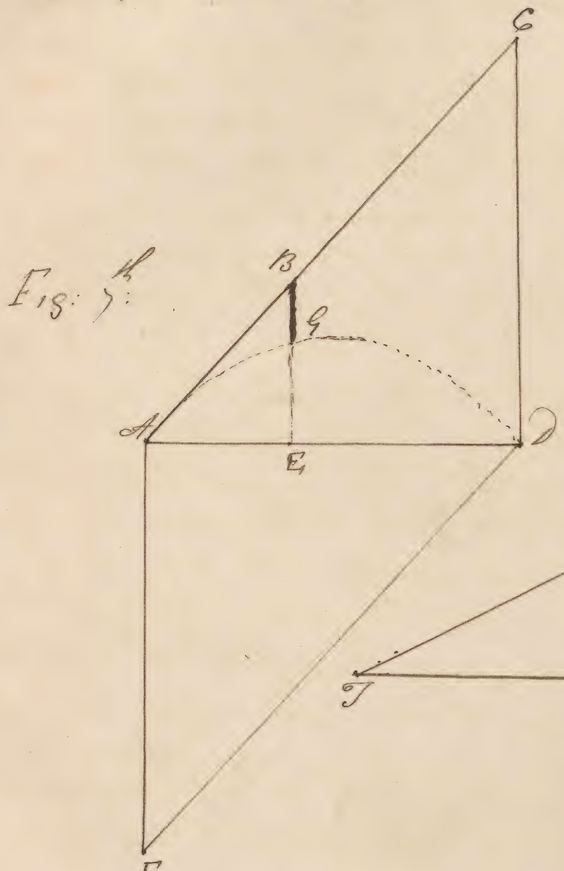


Fig: 5.

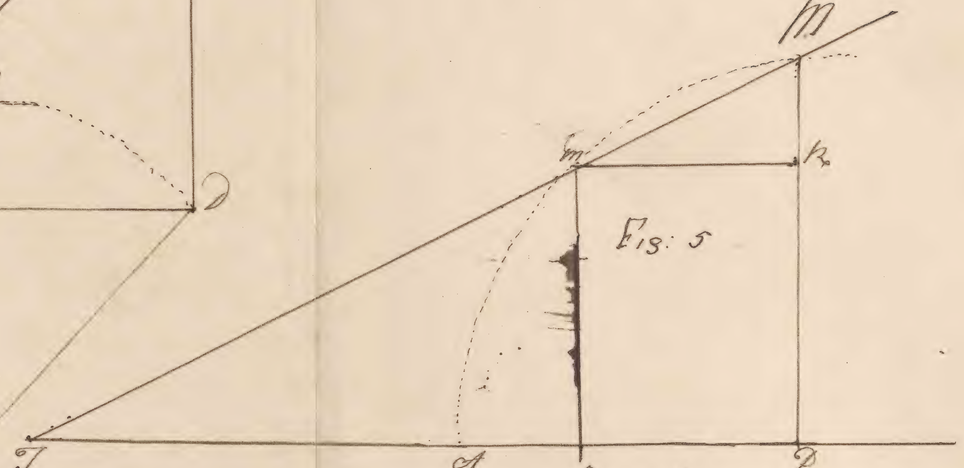


Fig: 5

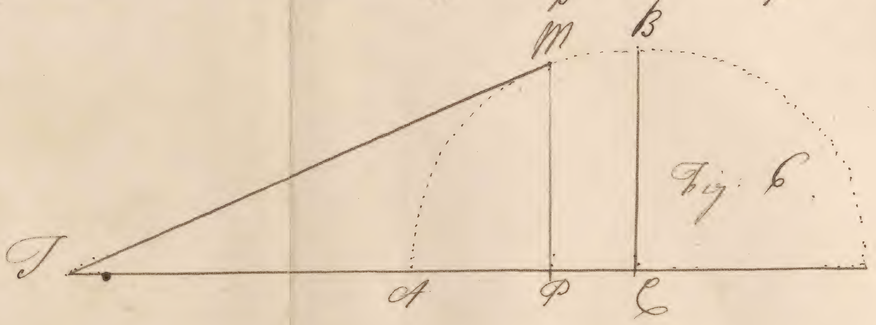


Fig: 6

that the Resistance of y^e Base BC is call'd
AB^q. But if this Resistance as Ding by the
supposition a constant Quantity be called s. the
Resistance of the frustum the Resistance of y^e
Struction will be $\frac{AB^q - 2AE \times ED + ED^2}{AB^2}$,
or $1 - \frac{2AE \times ED + ED^2}{AB^2}$ and therefore when this
Quantity, supposing DE & EB constant, as
in the Problem, is a maximum or minimum, the
Resistance of the frustum will be so too. Call
DE, a, BE, b, AE, x, & we have $1 - \frac{2AE \times ED + ED^2}{AB^2} =$
$= 1 - \frac{2ax + aa}{xx + bb}$ whose Fluxion $\frac{2aaxx - 2aaxx - 2abb\dot{x}}{xx + bb^2}$
in our Case $= 0$. whence $xx - ax - bb = 0$ & $x =$
$\frac{1}{2}a \pm \sqrt{\frac{aa}{4} + bb}$. But x the height of y^e whole
Cone must be greater than a which is the height
of but part of y^e Cone, therefore $x = \frac{1}{2}a + \sqrt{\frac{aa}{4} + bb}$
whence we have the following Construction, since
BC y^e Diameter of the greate Base & DE y^e height
of y^e frustum are both given, bisect DE in H
draw HC & produce ED to K so y^t HK may
be equal to HC. Then will y^e Triangle CBK
represent a Cone whose y^e frustum BDEC
shall meet with y^e least Resistance in an uniform
fluid, than any other of y^e same Base
& Altitude. Q E D

That this Resistance is a minimum & not a
Maximum is evident from hence, y^t the
sign of the fluxion of this Resistance

2 Problems relating to the Frustums of least Resistance. ——

1.— Let the Diameter of y^e Base of a Frustum of a Cone be 8 Inches & its height 6, w.^t must the Diameter of the lesser Base be to make it a frustum of least Resistance, or which is the same thing, what must be the Height of y^e Cone, out of which this frustum of least Resistance is to be cut? Ans.^r 8 Inches. And: A 2(8) : C B(8) :: A D(2) : G F(2) Inches, the Diameter of the lesser Base.

2. What Proportion will the Resistance of y^e Frustum bear to y^e resistance of a Cone of y^e same Base and Height. If I represents the Resistance of the greater Base B C the Resistance of a Frustum, B F G C will be $\frac{B2 + D^2}{AB^2}$ as is evident from y^e former Computation. Let now A come to D & the Frustum B F G C will now be changed into a Cone of y^e same base & Altitude, which Cone will be represented by B D C & its Resistance by $\frac{BD^2}{BD^2}$ therefore the Resistance of y^e frustum will be to the Resistance of a Cone of the same Base & height as $\frac{BD^2 + AD^2}{AB^2} : \frac{BD^2}{BD^2}$. that is in our Case as $\frac{1}{4}$ to $\frac{4}{13}$ or as 13 to 16 —

depends upon the sign of the Quantity xx — ax — bb. which when x = a is negative & therefore the resistance of the Frustum BFGC will be less and less till the Cone ABC whereof it is a frustum, has obtained the Height already assigned.

Cor. 1. Therefore the frustum BFGC meets w.th less Resistance y.n a Cone of y.e same base & Altitude —

Cor. 2. The Resistance of a Frustum BFGC will be to the Resistance of any other Frustum of y.e same Base & Altitude, as NB^2 is to $NB^2 + \frac{Dd}{2A} \times NA^2$ supposing N to be the Vertex of the Cone whereof BFMC is a Frustum. — —

Examples, shewing the Use of y^e Binomial Theorem

Sup: it be required to raise $p+q$ to y^e 10.th power. By y^e help of y^e Theorem. we must first lay out as is called y^e Series & then compute the Terms. thus;

1.st $p+q^{10} = p^{10} + \frac{10}{1}\frac{Aq}{p} + \frac{9}{2}\frac{Bq}{p} + \frac{8}{3}\frac{Cq}{p} + \frac{7}{4}\frac{Dq}{p} + \frac{6}{5}\frac{Eq}{p} + \frac{5}{6}\frac{Fq}{p}$
$+ \frac{4}{7}\frac{Gq}{p} + \frac{3}{8}\frac{Hq}{p} + \frac{2}{9}\frac{Iq}{p} + \frac{10}{10}\frac{Kq}{p}$ Then compute y^e value of each Term & you have —

2.d — $p+q^{10} = p^{10} + 10p^9q + 45p^8q^2 + 120p^7q^3 + 210p^6q^4 + 225p^5q^5$
$+ 210p^4q^6 + 120p^3q^7 + 45p^2q^8 + 10pq^9 + q^{10}$ — — —

NB. You need not lay down or compute any farther. than till they return the Contrary in their Coefficients & their Indices begin to change. Thus.

$p+q^9 = p^9 + \frac{9Aq}{p} + \frac{9}{1}\frac{Aq}{p} + \frac{8}{2}\frac{Bq}{p} + \frac{7}{3}\frac{Cq}{p} + \frac{6}{4}\frac{Dq}{p} + \frac{5}{5}\frac{Eq}{p}$
$+ \frac{4}{6}\frac{Fq}{p}$ &c $\frac{r}{p}$

$p+q^9 = p^9 + 9p^8q + 36p^7q^2 + 84p^6q^3 + 126p^5q^4 + 126p^4q^5$
$+ 84p^3q^6 + 36p^2q^7 + 9pq^8 + q^9$ — — —

Again — $\overline{1-z}^8 = 1 - \frac{8}{1}Az - \frac{7}{2}Bz - \frac{6}{3}Cz - \frac{5}{4}Dz - \frac{4}{5}Ez - \frac{3}{6}$
$Fz - \frac{2}{7}Gz - \frac{1}{8}Hz$ /.

$\overline{1-z}^8 = 1 - 8z + 28z^2 - 56z^3 + 70z^4 - 56z^5 + 28z^6 - 8z^7$
$+ z^8$ —

In the same manner the following Examples are computed. Thus —

$$\frac{1}{z-1} = \left\{ z^2 - \frac{1}{1} A z - \frac{6}{2} B z - \frac{5}{3} C z - \frac{4}{4} D z - \frac{3}{5} E z - \frac{2}{6} F z - \frac{1}{7} G z \right.$$

$$\frac{6}{p+q} = \begin{cases} p^6 + \frac{6}{1} A q p^5 + \frac{5}{2} B q^2 p^4 + \frac{4}{3} C q^3 p^3 + \frac{3}{4} D q^4 p^2 + \frac{2}{5} E q^5 p + \frac{1}{6} F q^6 \\ p^6 + 6 p^5 q + 15 p^4 q^2 + 20 p^3 q^3 + 15 p^2 q^4 + 6 p q^5 + q^6 \end{cases}$$

$$\frac{5}{p-q} = \begin{cases} p^5 - \frac{5}{1} A q p^4 - \frac{4}{2} B q^2 p^3 - \frac{3}{3} C q^3 p^2 - \frac{2}{4} D q^4 p - \frac{1}{5} E q^5 \\ p^5 - 5 p^4 q + 10 p^3 q^2 - 10 p^2 q^3 + 5 p q^4 - q^5 \end{cases}$$

$$\frac{4}{p-q} = \begin{cases} p^4 - \frac{4}{1} A q p^3 - \frac{3}{2} B q^2 p^2 - \frac{2}{3} C q^3 p - \frac{1}{4} D q^4 \\ p^4 - 4 p^3 q + 6 p^2 q^2 - 4 p q^3 + q^4 \end{cases}$$

$$\frac{3}{p+q} = \begin{cases} p^3 + \frac{3}{1} A q p^2 + \frac{2}{2} B q^2 p + \frac{1}{3} C q^3 \\ p^3 + 3 p^2 q + 3 p q^2 + q^3 \end{cases}$$

$$\frac{1}{p-q} \begin{cases} = p' - \frac{1}{1} A q \\ p' - q \end{cases}$$

$$\frac{0}{p+q} = \begin{cases} p^0, \\ 1, \end{cases}$$

$$\frac{1}{1-z} = \frac{-1}{1-z} = \begin{cases} 1 + Az + Bz + Cz + Dz + Ez \&c \\ 1 + z + zz + zzz + z^4 + z^5 \&c \infty \end{cases}$$

That $\frac{1}{1-z} = \frac{1}{1-z}$ is plain from y^e observations on powers and Indexes thus y^e Index of $1-z$ is 1 its Recip: $\frac{1}{1-z}$ ch. is therefore equal to $\frac{-1}{-1}$. That $\frac{-1}{1}$ or $1^{-1} = 1$ is proved thus. 1^{-1} or $1^{-2} = \frac{1-z}{1}$ or $\frac{1}{1}$ but $\frac{1}{1}$ or $\frac{2}{1} = 1$ therefore $\frac{-1}{1}$ or $\frac{-2}{1} = 1$ in the same manner $1^{\frac{1}{2}} = 1^{-\frac{1}{2}} = 1$ Now the Value of $\frac{-1}{1-z}$ or $\frac{1}{1-z}$ might have been found by dividing 1 by $1-z$ thus.

$1-z)\,1.*.*.*.*.(1+z+zz+zzz$ &c as before.—

$1-z$

z

$z-zz$

zz

$zz-zzz$

$+zzz$

In this Division it is easier to use plain Division than the series, but when the powers of $1+z$ run high we must then be obliged, to have recourse to y^e series.

$\frac{1}{1+z^2}$ or $\frac{1}{1+2z+zz}$ or $\frac{-2}{1+z} = \begin{cases} 1-2Az-\frac{3}{2}Bz-\frac{4}{3}Cz-\frac{5}{4}Dz \\ \&c-2z+3z^2-4z^3+5z^4-6z^5 \&c. \end{cases}$

$\sqrt{1+z}$ or $1+z^{\frac{1}{2}} \begin{cases} 1+2Az-\frac{1}{4}Bz-\frac{3}{6}Cz-\frac{5}{8}Dz-\frac{7}{10}Ez \\ 1+\frac{1}{2}z+\frac{1}{8}z^2+\frac{1}{16}z^3+\frac{5}{128}z^4+\frac{7}{256}z^5 \&c. \end{cases}$

This series might have been produced by extracting the sq: root of $1+z$. thus

$1+z$ $\left(1+\frac{1}{2}z-\frac{1}{8}zz+\frac{1}{16}z^3-\frac{5}{128}z^4 \&c.\right.$

1

$2+\frac{1}{2}z)\,z$

$\frac{1}{2}z \quad z+\frac{1}{4}zz$

$2+z-\frac{1}{8}zz)\,*-\frac{1}{4}zz$

$-\frac{1}{8}zz \quad -\frac{1}{4}zz-\frac{1}{8}z^3+\frac{1}{64}z^4$

$2+z-\frac{1}{4}z^2+\frac{1}{16}z^3)\,*\quad +\frac{1}{8}z^2-\frac{1}{64}z^4$

$+\frac{1}{16}z^3)$

$+\frac{1}{8}z^3+\frac{1}{16}z^4 \;|\; \frac{1}{64}z^5+\frac{1}{256}z^6$

$2+z-\frac{1}{4}z^2+\frac{1}{8}z^3-\frac{5}{128}z^4)\,*=\frac{5}{64}z^4+\frac{1}{64}z^5-\frac{1}{256}z^6$

$-\frac{5}{128}z^4) \quad -\frac{5}{64}z^4+\frac{5}{128}z^5+\frac{5}{512}z^6-\frac{25}{1024}z^7+\frac{25}{16384}z^8$

$* \;|\; +\frac{7}{128}z^5-\frac{7}{112}z^6+\frac{5}{1024}z^7-\frac{25}{16384}z^8$

29

This is the work at large. but since no more than 5 terms are wanting, you may may omit all y^e powers of x above x^4 & y^e work will be much contracted as will appear by comparing it with the work at large, where you'l find all the terms on the right hand the Line are omitted; an Example & proof whereof is as follows.

$$1+z \;*\;*\;*\;* \;\Big(1+\tfrac{1}{2}z-\tfrac{1}{8}z^2+\tfrac{1}{16}z^3-\tfrac{5}{128}z^4$$

$$2+\tfrac{1}{2}z \;\Big)\; \overline{*+z} \qquad 1$$
$$+z+\tfrac{1}{4}z^2$$
$$2+z-\tfrac{1}{8}z^2 \;\Big)\; \overline{*-\tfrac{1}{4}z^2+*\;*} \qquad +\tfrac{1}{2}$$
$$-\tfrac{1}{4}z^2-\tfrac{1}{8}z^3+\tfrac{1}{64}z^4$$
$$2+z-\tfrac{1}{4}z^2+\tfrac{1}{16}z^3 \;\Big)\; \overline{*\;+\tfrac{1}{8}z^3-\tfrac{1}{64}z^4+*\;*} \qquad -\tfrac{1}{8}$$
$$+\tfrac{1}{8}z^3+\tfrac{1}{16}z^4$$
$$2+z-\tfrac{1}{4}z^2+\tfrac{1}{8}z^3-\tfrac{5}{128}z^4 \;\Big)\; \overline{*-\tfrac{5}{64}z^4} \qquad +\tfrac{1}{16}$$
$$-\tfrac{5}{64}z^4$$
$$\overline{*}$$

The same contraction may be used in Multiplication, as in 4° following Example, which is a proof of the former Division.

$$1+\tfrac{1}{2}z-\tfrac{1}{8}z^2+\tfrac{1}{16}z^3-\tfrac{5}{128}z^4$$
$$1+\tfrac{1}{2}z-\tfrac{1}{8}z^2+\tfrac{1}{16}z^3-\tfrac{5}{128}z^4$$
$$1+\tfrac{1}{2}z-\tfrac{1}{8}z^2+\tfrac{1}{16}z^3-\tfrac{5}{128}z^4$$
$$+\tfrac{1}{2}z+\tfrac{1}{4}z^2-\tfrac{1}{16}z^3+\tfrac{1}{32}z^4$$
$$-\tfrac{1}{8}z^2-\tfrac{1}{16}z^3+\tfrac{1}{64}z^4$$
$$+\tfrac{1}{16}z^3+\tfrac{1}{32}z^4$$
$$-\tfrac{5}{128}z^4$$

$$1+z \;*\;*\;*$$

Lemma Of Infinite Series —

If $A + Bz + Cz^2 \&c = a + bz + cz^2$. whatever Value is put upon z. then will $A = a$ $B = b$, $C = c$ for since in all Cases $A + Bz + Cz^2 = a + bz + cz^2$ let $z = o$ & we shall have $A = a$ therefore $Bz + Cz^2 = bz + cz^2$ & $B + Cz = b + cz$ universally. Let z again $= o$ & we shall have again $B = b$ wherefore $Cz = cz$ & $C = c$ & &c — —

Theorem.

$p + q^m = p^m + \frac{m}{1} \frac{Aq}{p} + \frac{m-1}{2} \frac{Bq}{p} + \frac{m-2}{3} \frac{Cq}{p} + \frac{m-3}{4} \frac{Dq}{p} + \frac{m-4}{5} \frac{Eq}{p}$ &c. where A denotes the first Term B the second C 3 &c. —

Demonstration

Supposing z to be any variable Quantity Let $p + qz^m = A + Bz + Cz^2 + Dz^3 + Ez^4$ &c without Restriction Then supposing $z = o$ we have $p^m = A$. therefore suppose $z = 1$ we have $p + q^m = p^m + B + C + D + E$. Again since $A + Bz + Cz^2 + Dz^3$ &c $= p + qz^m$ we have $Bz + 2Cz\dot{z} + 3Dzz\dot{z} + 4Ez^3\dot{z} = mqz\dot{z} \times \overline{p + qz^{m-1}}$ therefore $B + 2Cz + 3Dz^2 + 4Ez^3 = \left(mq \times \overline{p + qz^{m-1}} \right.$ $\left. = mq \times \frac{p + qz^m}{p + qz^3} \right) = \frac{mqA + mqBz + mqCz^2 + mqDz^3}{p + qz^3}$ therefore $mqA + mqBz + pmqCz^2 + mqDz^3$ &c $= Bp + 2Cpz + Dpz^2 + mqAz^3 + 2mqCz^3 + 3mqDz^4$ &c. therefore by the Lemma $mqA = Bp$. $mqB = 2Cp + Bq$. $mqC = 3Dp + 2Cq$. $mqD = 4Ep + 3Dq$ &c. which Equations in their different order give $\frac{m}{1} Aq = Bp$. $\frac{m-1}{2} Bq = Cp$. $\frac{m-2}{3} Cq = Dp$. $\frac{m-3}{4} Dq = Ep$ &c. therefore $p + q^m = p^m + \frac{m}{1} \frac{Aq}{p} + \frac{m-1}{2} \frac{Bq}{p} + \frac{m-2}{3} \frac{Cq}{p} + \frac{m-3}{4} \frac{Dq}{p} + \frac{m-4}{5} \frac{Eq}{p}$ &c 2 &c. — — —

In this Step observe that you find the fluents of both sides y^q Equation. To find therefore the fluent of $\dot{y}\dot{z} \times p + qz^m$ make $p + qz = y$ & you'll have $\dot{y}z = \dot{y} \& \dot{q}x \times p + qx^m = \dot{y} \times y^m$ But the fluent of $\dot{y} \times y^m = \frac{y^{m+1}}{m+1}$ therefore the fluent of $\dot{q}z \times p + qz^m$ is $\frac{p + qz^{m+1}}{m+1}$ ——

Again the fluent of y^q other side of y^q Equation. viz $A\dot{q}z + B\dot{q}zz$ &c will be found by the following Problem to be $Aqz + \frac{1}{2}Bqz^2$ but before we can reason an equality of fluents from an equality of fluxions we must compare $\frac{p+qz^{m+1}}{m+1}$ & $Aqz + \frac{1}{2}Bqz^2$ &c when $z = 0$. Now in this case, we must add $\frac{p^{m+1}}{m+1}$ to $Aqz + Bqz^2$ &c & then we have $\frac{p+qz^{m+1}}{m+1} = \frac{p^{m+1}}{m+1} + Aqz + Bqz^2$ &c. — —

~~$\frac{p+qz^{m+1}}{m+1}$~~ $Aqz + \frac{1}{2}Bqz^2$ &c $= 0$ but $\frac{p+qz^{m+1}}{m+1}$ at y^e time will be equal to $\frac{p^{m+1}}{m+1}$ that the fluents may be equal to one another. — — — —

From the 3^d Coral. it appears y^t of y^e two Quantitys p & q. the greater ought always to be set first. when it is indifferent which is first, as is when any power of $p+q$ or any even integral power of $p-q$ is sought, for then $p+q^m = q+p^m$ & m being an even number $p-q^m = q-p^m$ otherwise the series will not Converge but diverge.

The Indices of y^e first Quantities, continually decreasing while the others increase; hence q always represents y^e lesser of y^e two Quantities $p+q$, & when any power of $1+z$ is sought, z is always supposed less than, 1 —

Otherwise thus.

Let $p + qz^m =$ &c as before. then $qz \times$

$p + qz^m = Aqz + Bqzz + Cqz^2z + Dqz^3z$ &c. therefore

$\frac{p+qz^{m+1}}{m+1} = \frac{pm+1}{m+1} + Aqz + \frac{1}{2}Bqz^2 + \frac{1}{3}Cqz^3$ &c as will

appear by comparing the fluents, when $z = o$. But

$$\frac{p+qz^{m+1}}{m+1} = p + qz \times \frac{p+qz^m}{m+1} = p + qz \times \frac{A + Bz + Cz^2 + Dz^3 \text{ &c}}{m+1} =$$

$$= Ap + Bp + Aq \times \frac{z + Cp + Bq}{m+1} \times zz + \frac{Dp + Cq}{m+1} \times z^3 + \frac{Dp + Dq}{m+1} \times z^4$$

therefore $\frac{Ap + Bp + Aq}{m+1}$ &c &c $= \frac{p^{m+1}}{m+1} + Aqz +$

$\frac{1}{2}Bqz^2 + \frac{1}{3}Cqz^3$ &c. $^{m+1}$ therefore by y$^{\text{e}}$ Lemma $\frac{Ap}{m+1} =$

$= \frac{pm+1}{m+1}$, $\frac{Bp + Aq}{m+1} = Aq \cdot \frac{Cp + Bq}{m+1} = \frac{1}{2}Bq \cdot \frac{Dp + Cq}{m+1}$

$= \frac{1}{3}Cq \frac{Dp + Dq}{m+1} = \frac{1}{4}Dq$ &c.

therefore $p^{\frac{m}{1}} = A \cdot \frac{m}{1} \frac{Aq}{p} = B \frac{m-1}{2} \frac{Bqq}{pp} = C$ &c &c

as above. ——

Cor. 1$^{\text{o}}$ $p + qz^{\frac{m}{n}} = p^{\frac{m}{n}} + \frac{m}{n}\frac{Aqz}{p} + \frac{m-n}{2n}\frac{Bqzz}{pp} + \frac{m-2n}{3n}$

$\frac{Cqz^3}{p^3}$ &c ——

Cor. 2. $r \times p + qz^{\frac{m}{n}} = rp^{\frac{m}{n}} + \frac{m}{n} \times \frac{Aqz}{p} + \frac{m-n}{2n}$

$\frac{Bqzz}{pp}$ &c where $A = rp^{\frac{m}{n}}$ &c ——

Cor. 3$^{\text{d}}$ — If $q = o$ the whole series will be comprehended in the first Term, therefore y$^{\text{e}}$ Index of z is in respect of p. the Index with the Letter Coefficient. cf. B. The latter Dem: of this Proposition cannot be well understood till after the following Problem. —

Prob. 5 —

To find the fluent of $p\dot{x}x^q$ fluxion in $\frac{\dot{x}}{x}$.

Suppose the fluent of $p\dot{x}x^q$ fluxion to be px^g:
then will $pg\dot{x}x^{g-1}\dot{x}$ x^m therefore $pg x^{g-1}$
$= 1 x^m$ therefore $pg = 1$ & $g-1 = m$ therefore
$g = 1+m$ & $p = \frac{1}{1+m}$ & the fluent sought,
will be $\frac{1}{1+m} x^{1+m}$

Cor. $\dot{x}x^{\frac{m}{n}}$ will have for its fluent $\frac{n}{m+n} x^{\frac{n+m}{n}}$.

Examples of finding the fluents of Fluxions —

1— what is y^2 fluent of $\dot{x}x$. In this Case $m=1$ & $n+1=2$ wherefore by 4th Prob. for finding fluents y^2 fluent sought is $\frac{1}{2}xx$.

2—of $2\dot{x}x$ is xx.

3—of $p\dot{x}$ is px

4—of $\frac{x\dot{x}}{2}$ or $\frac{1}{2}\times x\dot{x}$ is $\frac{xx}{4}$

5- of $\frac{\dot{x}}{x^3} = 1\times\dot{x}\times x^{-3}$ is $\frac{1}{2}x^{-2}$ or $\frac{1}{2}\frac{1}{xx}$ or $\frac{1}{2xx}$ —

6 of $\frac{\dot{x}}{xx} = 1\times\dot{x}\times x^{-2}$ is $1\times\frac{1}{-1}x^{-1}$ or $\frac{-1}{x}$.

7 of $\sqrt{xx} = xx^{\frac{1}{2}}$ is $\frac{2}{3}x^{\frac{3}{2}}$ for by Coral. $\frac{n+m}{n} = \frac{3}{2}$ & $\frac{n}{n+m}$ $= \frac{2}{3}$ wherefore the fluent is $\frac{2}{3}x^{\frac{3}{2}}$. . .

8. of $\frac{\dot{x}}{\sqrt{x}} = \dot{x}\times x^{-\frac{1}{2}}$ is $2x^{\frac{1}{2}}$ or $2\sqrt{x}$ —

9. of $xx\sqrt{x}\dot{x} = x^{\frac{5}{2}}\dot{x}$ is $\frac{2}{7}x^{\frac{7}{2}}$ —

10 of $\frac{x\dot{x}\sqrt{x}}{\sqrt{x}} = \dot{x}\times x^{\frac{1}{6}}$ is $\frac{1}{7}x^{\frac{7}{1}}$. of $\dot{x}\frac{\sqrt[2]{x}}{\sqrt[3]{x}}$

11 of $\dot{x}\frac{\sqrt[3]{x}}{x} = \dot{x}\times x^{\frac{-2}{3}}$ is $3x^{\frac{1}{3}}$ or $3\sqrt[3]{x}$.

12 of $\frac{\dot{x}}{x} = x\times x^{\frac{-1}{1}\frac{1}{0}}x^{\frac{0}{1}}$ w^t is infinite —

13 of $\dot{x}\sqrt{1+x}$ put $1+x=\sqrt{1+x}$ & $\zeta\zeta = 1+x$ & $2\zeta\dot{\zeta} = \dot{x}$ & $\dot{x}\sqrt{1+x} = 2\zeta\dot{\zeta}$ whose fluent is $\frac{2}{3}\zeta^3 = \frac{2}{3}\zeta^2\times\zeta = \frac{2}{3}1+x\times\sqrt{1+x}$ $= \frac{2}{3}\times 1+x^{\frac{3}{2}}$ wherefore the fluent of $\dot{x}\sqrt{1+x}$ $\frac{2}{3}1+x^{\frac{3}{2}}$. —

14 of $\frac{xx\dot{x}}{\sqrt{rx}} = r^{-\frac{1}{2}}\times x^{\frac{3}{2}}\times\dot{x}$ is $\frac{2}{5}r^{-\frac{1}{2}}x^{\frac{5}{2}} = \frac{2}{5}\frac{\sqrt{x^5}}{\sqrt{r}}$ —

15 of $\dot{x}\sqrt{2rx} = 2r^{\frac{1}{2}}\times x^{\frac{1}{2}}\times\dot{x}$ is $\frac{2}{3}\sqrt{2rxxx}$ —

16 of $\dot{x}\sqrt{ax-aa}$ put $\zeta = \sqrt{ax-aa}$ & we shall have $\zeta\zeta = ax-aa$ & $2\zeta\dot{\zeta} = a\dot{x}$ & $2\zeta\zeta\dot{\zeta} = a\dot{x}\sqrt{ax-aa}$ whose Fluent will be $\frac{2}{3}\zeta^3 = \frac{2}{3}ax-aa\times\sqrt{ax-aa} = \frac{2ax-2aa}{3}\times$ $\sqrt{ax-aa}$. ——

17. of $\frac{x\dot{x}-x\dot{x}}{\sqrt{2rx-xx}}$ put $\zeta = \sqrt{2rx-xx}$ & we shall have $2rx-xx$ $= \zeta\zeta$ & $2r\dot{x}-2x\dot{x} = 2\zeta\dot{\zeta}$ & $r\dot{x}-x\dot{x} = \zeta\dot{\zeta}$ & $\frac{r\dot{x}-x\dot{x}}{\sqrt{2rx-xx}} =$ $\dot{\zeta}$ whose fluent is $\zeta = \sqrt{2rx-xx}$ — —

If $\frac{\dot{x}x}{\sqrt{rr+xx}}$ make $\sqrt{rr+xx}=y$ & then $rr+xx=yy$ & $2x\dot{x}$
$=2y\dot{y}$ & $x\dot{x}=y\dot{y}$ therefore the fluent of $\frac{x\dot{x}}{\sqrt{rr+xx}}$ = the
fluent of $\dot{y}$, but the fluent of $\dot{y}$ is y or $\sqrt{rr+xx}$ therefore the fluent of $\frac{x\dot{x}}{\sqrt{rr+xx}}=\sqrt{rr+xx}$. And by
this Method may the fluents of such Quantities be often found but where y^e method fails, there is
no other way of finding them but by drawing them
into an Infinite Series, of wch follow an Example
or 2 — — —

1. What is y^e fluent $\frac{\dot{x}}{1+x}=\dot{x}\frac{1}{1+x}$.
First $1+x$ must be resolved into an Infinite Series
either by Division or y^e Binomial Theorem, wch
will give $\frac{1}{1+x}=1-x+xx-x^3+x^4$ &c whence
$\dot{x}\times\frac{1}{1+x}\dot{x}-x\dot{x}+x^2\dot{x}-x^3\dot{x}$ &c. Now you may
easily find y^e fluent of each Term in y^e Series
whence you will get an Infinite Series equal
to y^e fluent sought (viz) fluent $\frac{\dot{x}}{1+x}=x-\frac{1}{2}x^2+$
$+\frac{1}{3}x^3-\frac{1}{4}x^4+\frac{1}{5}x^5$ &c — —

2. What is the fluent of $\frac{\dot{x}}{\sqrt{1\pm xx}}=\dot{x}\times 1\mp\frac{1}{2}xx$ —
This being thrown into an Infinite Series we
shall get $\frac{\dot{x}}{\sqrt{1-xx}}=\dot{x}\times$ this Series $1+\frac{1}{2}xx\dot{x}+\frac{3}{8}x^4\dot{x}$
$+\frac{5}{16}x^6\dot{x}+\frac{35}{128}x^8\dot{x}$ &c. In finding the fluent
of each term we shall have fluent $\frac{\dot{x}}{\sqrt{1-xx}}=$
$x+\frac{1}{6}x^3+\frac{3}{40}x^5+\frac{5}{112}x^7+\frac{35}{1152}x^9$ &c.

Now to shew y^t this Series is Regular, let us
consider by what Quantity each term must be multi-
plied in order to produce the following, & if these
Factors be found regular the Series must be so to —

& its Law (as he's called) uniform. To find these factors we must divide each Term by y^e next preceding & y^e shall have the series of factors sought, viz. $\frac{1}{6}x^2 + \frac{9}{20}x^2 + \frac{25}{42}x^2$ $+ \frac{49}{72}x^2$ &c. y^is the first Term x multiplied by $\frac{1}{6}x^2$ produces y^e 2^d Term. the 2^d x $\frac{9}{20}x^2$. gives y^e 3^d &c now in this series of factors the Coefficient x^2 is always the same & therefore regular. the difference of y^e Terms being only in their Coefficients. in w^ch we may observe this Regularity that their numerators are a series of y^e squares of y^e odd numbers, & their Denominators a series of every even number multiplied into y^e odd one next succeeding it, whence if we extend the series by this way, we shall have y^e fluxion of $\frac{x}{\sqrt{1+xx}} = x - \frac{1\times1}{2\times3}Ax^2 + \frac{3\times3}{4\times5}Bx^2 + \frac{5\times5}{6\times7}Cx^2 + \frac{7\times7}{8\times9}Dx^2$ &c now this series appears regular, and we have y^e advantage among others of this method, which y^r Inventor uses. If we can see the Law of y^e Series & continue the Terms as far as we please without any Trouble but y^e of writing it out. —

In y^e same manner may we find y^e Law of any series by finding y^e of the Series of their Factors. & if it does not appear y^t y^e series is regular, we may proceed to find a third Series, whose Terms are the Factors producing y^e 2^d Series & so on— Let us apply to find the Law of y^e Series produced in raising a Binomial to any Power & thus by a very gen: Induction, prove (tho' not demonstrate) Newtons Theorem.

Raise therefore any Binomial Quantity as $p+q$ to any power so high as to produce a number of

Terms, sufficient for y^e Observation, as y^e 6^th thus
$p+q^6 = p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 +$
$+ 6pq^5 + q^6$ Proceeding each Term by the
next preceding one we have y^e series $\frac{6q}{p} + \frac{5q}{2p} +$
$\frac{4q}{3p} + \frac{3q}{4p} + \frac{2q}{5p} + \frac{1q}{6p}$ whence if A^t the first
Term be^t called A, the 2^d will be $\frac{6}{1}Aq$ which
being called the 3^d will be $\frac{5}{2}\frac{6}{p}Bq$ that is $p+q^6 =$
$p^6 + \frac{6}{1}Ap^5q + \frac{5}{2}Bp^4q + \frac{4}{3}Cp^3q$ &c. now its easy to see
that y^e numerators of y^e Coefficients decrease in every
Term by the Constant number 1, the numerator
in y^e first Term being $= 6$ y^e Index; but the
Denominators increase in y^e same manner; y^e
Denom. in y^e 1^st Term whose A is concerned or y^e 2^d in
order being 1. but y^e first member of y^e series
or p^6 is such a power of y^e first Quantity as y^e
Index denotes but this is true whatever
be y^e Index, therefore if we put m universally
for y^e Index the Laws will stand thus (vid)
$p+q^m = p^m + \frac{m}{1}Ap^{m-1}q + \frac{m-1}{2}Bp^{m-2}q$ &c
which is Sr Isaac Newtons Theorem —
Tis not improbable that the Laws of most
series were first found out this way &
then demonstrated by other means —

37

Before we proceed to the next Chap: it must be observed that no flowing quantity, as x has above, one fluxion viz ẋ But tho the same fluxion may have an infinite number of fluents, as x a+x, b+x, &c provided a, b, c be invariable Quantities because the fluxion of all these are ẏ same with ẋ, nor can their Multiplicity of fluents create any Confusion in Calculation. because we never have occasion to find the fluent of any fluxion or v. v. but in order to compare it with some other fluxion, or fluents now there can be no difficulty in pronouncing an Equality of Fluxions but before we pronounce an Equality of fluents, we must compare them in y^e most simple known Case & if we then find them Equal we may safely pronounce y^m equal in all other Cases, But if in the simpler Case we find them not equal we must then make them equal by Addition or subtraction of some constant Quantity that, they may be universally so. As for Example if not x & y be two flowing Quantities always equal to each other, then we may safely conclude that ẋ & ẏ will be always equal. But on y^e other hand y^e fluxion of x be always equal to ẏ, we cant immediately conclude that x will be always equal to y. we must therefore assume some simple Case as y^t when x = 0 if we find that from the nature of the thing y = 0, we may safely conclude in gen: that x = y. because it was so in the Case assumed. But if when x = 0 y should = a, y^t a must be added to x & we may conclude universally that a + x = y. because it is in y^e Case assumed. Therefore the fluent of ẋ is not x but a + x

and these precautions must be always usd. if we would avoid Error in our Conclusions. — This may be illustrated by the following Instance.

If two Bodies B. we will suppose always in motion be constantly and at all times found to have run thro' y^e same space. it is certain that at every Instant of time they will both have the same Velocity. however irregular their motions may be — But on y^e contrary tho' we shd suppose two such Bodies at every instant of time to have the same Velocity, it does not follow y^t they have moved through equal Spaces unless it be premis'd that at some Instant they had moved thro' equal Spaces. — — —

These things being premised it will be proper to 2^d the 2^d Demonstration of Newtons Binomial Theorem, in 4^o Chap. of Infinite Series & then to the Quadrature of Curves —

On the Quadrature of Curves

To prove that $y\dot{x}$ is the true fluxion of the Area of a Curve whose Abscis is x & Ordinate y.

Let the Line BC be supposed to move uniformly and parallel to itself along the Line AB, then if the Point C be Relatively at rest in y^e^ Line BC, that Line BC will Describe a Parallelogram. And as the Line BC is supposed to flow uniformly upon AB so y^e^ Area of y^e^ Parallelogram will flow uniformly, Let us now suppose the Point C to move along the Line BC with an accelerated Velocity, therefore the fluxion or velocity of this Area cannot be better found, than by taking of the cause of y^e^ Acceleration for a time. Thus if we suppose BC in the Position DM, & C do not take the cause of acceleration, y^t^ is the motion of y^e^ Point C when B comes into D, C will come into m., & then the Increment of the Area will be DMmp. But if in order to find the fluxion of the Area we stop the motion of C, when BC comes into DM. then will B come into D & C into h & therefore the true fluxion of the Area will be Parallelogram ~~illegible~~ DMhm. because this is the Increment of the Area gotten with the uniform Velocity it had in the Point B

Quadrature of Curves

Prob. 6th

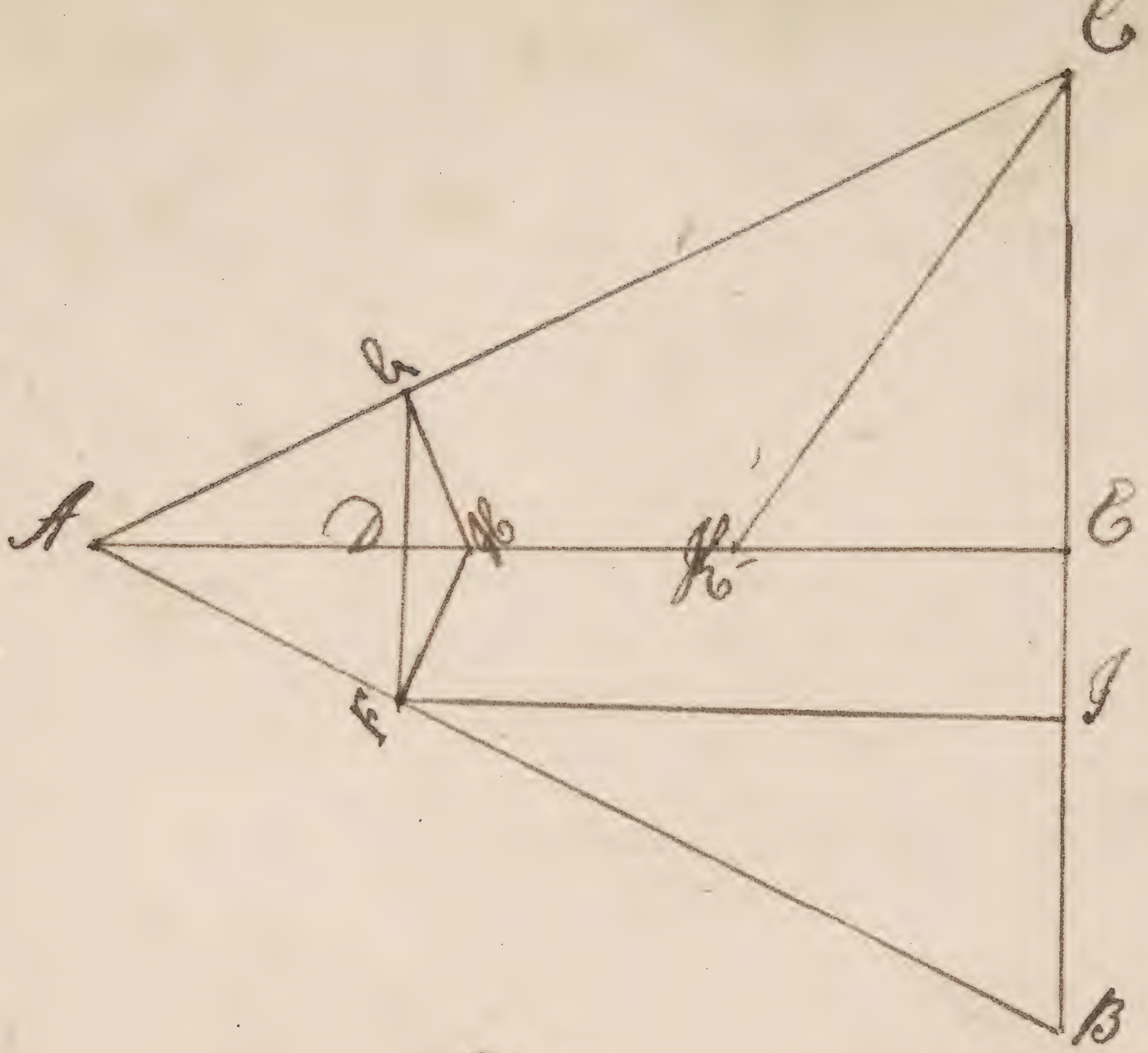
{6 Fig 3. Tab. 2}

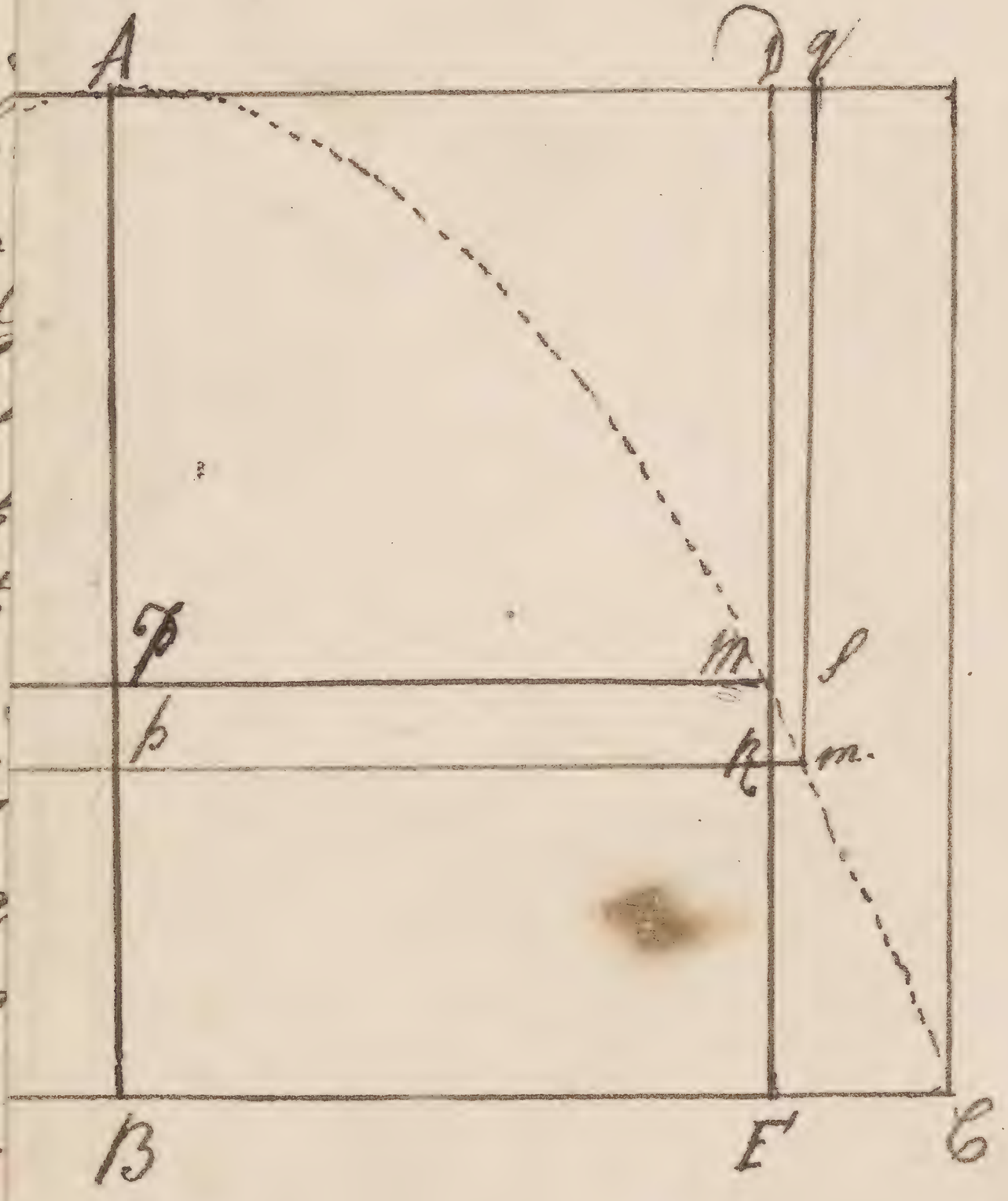
Let Am be any Curve whereof AP is the Axis & MP a perpendicular ordinate it's required to square or find the Area of y^e Curvilinear Space APM.

Solution —

Suppose another Ordinate as mp, parallel & infinitely near to PM & compleat y^e Parallelogram PRpQ then will the inscribed Parallelogram be to y^e circumscribed one, i.e. Ph : pd :: PM mp, & cons. in the present case in a Ratio of Equality. Therefore the Parallelogram Ph will be to the curvilinear Space PMmp more especially in a Ratio of Equality. & therefore in y^e Case where PR & Qp coincide the Parallelogram and the Curvilinear Space, may be used for each other.

Call now AP x, MP y & we shall have pd = ẋ & the Curvilinear Space PMmp. or the Parallelogram Ph will be yẋ. But y^e Curvilinear is y^e infinitely small increment or fluxion of y^e Area APM. therefore if the fluent of y^e fluxion be found which must be had from y^e known nature of y^e Curve such a fluent will exhibit the Area sought; especially if it begins to flow with the Area it self but if it does not the beginning must be cut off, or the defect supplyed by the Addition or subtraction of a known Quantity, & then the fluent will be equale to the Area sought.


C
b
A
D
K
C
Fig: 2
F
g
B


A
D
q
p
m
l
b
k
n.
B
E
6

Example 1st —

Let AM be the common Parabola, whose parameter ℄ be p. & we shall have from the nature of the Curve. $px = yy$ & $y\dot{y} \& \dot{y} = \sqrt{p} \times \sqrt{x}$. & $y\dot{x} = \sqrt{p} \times \dot{x} \times \sqrt{x}$ whose fluent is $x \cdot \frac{2}{3}\sqrt{p} \times x^{\frac{3}{2}}$ or $\frac{2}{3}\sqrt{p} \times \sqrt{x} \times x$ or $\frac{2}{3}xy$. Therefore if $2^{da} \frac{2}{3}xy$ flo with the same velocity w^ch the Area itself. But y^e 2^da $\frac{2}{3}xy$ is nothing when $x = 0$ & y^e same may be pronounced of y^e Area of A Par. therefore y^e fluent and the Area begin to flow together. & consequ^tly Area will be equal to y^e fluent, i.e. the Area A will be $\frac{2}{3}$ AP × PM or $\frac{2}{3}$ of the Circumscribed parallelogram PZ. It may not be improper to shew one for all, y^t instead of making $px = yy$ we had supposed one $x \times x = yy$ & instead of making AP the Axis we had supposed it any other diame of y^e Parabola AM. the Proportion between the Area AM. & the Parallelogram PZ would still have been the same. For however the several Areola PM kp. would have been chan by such a Proportionable Alteration of the equal Inclinations of y^e Ordinates PM. the same proportionable change. & that for the sa Reason would have happened & w^t for the sa Reason to y^e Parallelogram PZ. — —

Ex: 2^da —

Let it now be required to measure the extema space A2M.

Since as before $p \times 2M = A2^2$. if we call A & 2My. we shall have $py = xx$ & $y = \frac{xx}{p}$, &

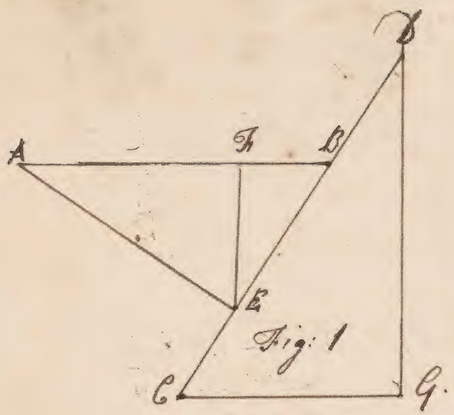


Fig: 1

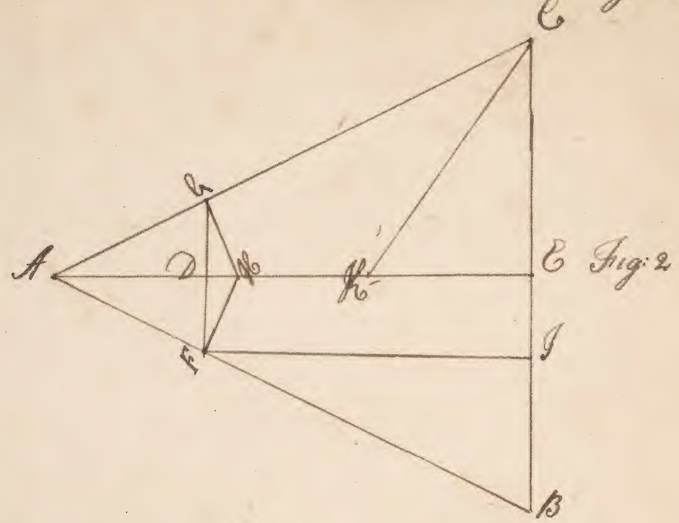


Fig: 2

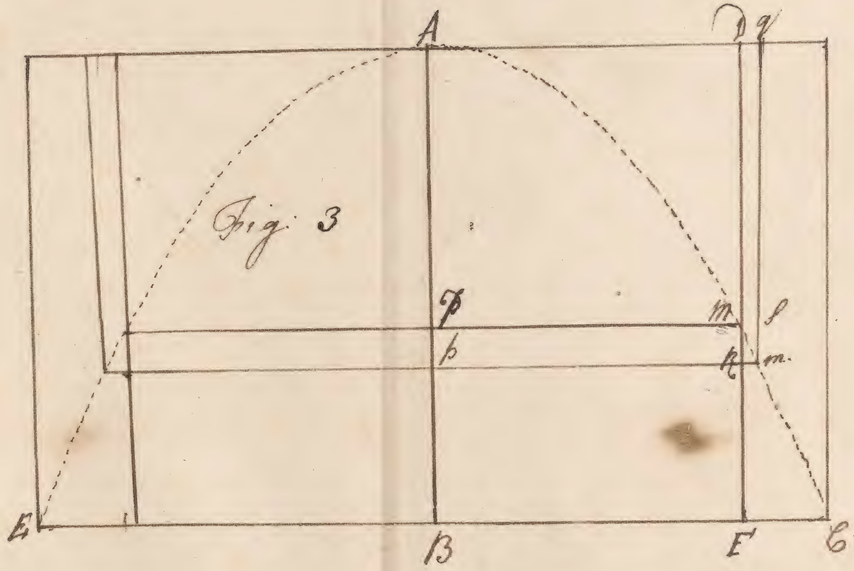


Fig. 3

& $y\dot{x} = \frac{xx\dot{x}}{p}$ whose fluent is $\frac{x^3}{p}$ or $x\frac{y}{3}$ will be the Area sought. so y^{e} the $\frac{3}{3}$ External Area ALM is $\frac{1}{3}$ part of the Parallelogram DL which confirms the former Example. —

Ex. 3^{d}

Let the Area of the Parabolic Segment CMB be required. Since $p \times AB = CB^2$ & $p \times AD = DM^2 =$ $= BJ^2 =$ Rectangle CJ &. Call now BC &. CJ. x JM, y. & we shall have, $py = 2bx - xx$ & $y = -2bx - xx$ & $y\dot{x} = \frac{2b\dot{x} - 2x\dot{x}}{p}$ whose fluent $bxx - \frac{1}{3}xxx$ will be the Area of y^{e} Segment sought. N.B. when x comes to AB we shall have $x = bC$ & the Parabolic Space CAB, will be found $= \frac{BC^3}{p} - \frac{1}{3}$ $\frac{BC^3}{p} = \frac{2}{3}\frac{BC^3}{p} = \frac{2}{3} AB \times \frac{BC^2}{p \times AB} = \frac{2}{3} AB \times BC$, which confirms the ~~following~~ former Example.

Ex. 4th

Let the nature of y^{e} Parabola be expressed by y^{e} general Equation $1 \times x^m = y^n$ here we have $y = x^{\frac{m}{n}}$ & $y\dot{x} = \dot{x} \times x^{\frac{m}{n}}$ whose fluent $\frac{n}{m+1} \times x^{\frac{m+n}{n}}$ or $\frac{n}{m+n} xy$ will be the Area sought of DM. therefore the Area ADM will be to the Parallelogram DL as n to $m+n$. — — —

Tab 3
Fig 1.
Pag 64

Ex. 5th

Let the Curve KG be referred to y^{e} Line AB by means of the Ordinate GM. parallel to AC & both perpendicular to AB & let the nature of the Curve be expressed by this Equation $AB \times DM^2 = 1$ then it's easy to see that y^{e} Lines AB & AC are Asymptotes to

the Curve & calling $\dot{x}$ D M, y. we shall have $y = \frac{1}{\sqrt{x}}$ & $y\dot{x} = \frac{\dot{x}}{\sqrt{x}}$ whose fluent is $2\sqrt{x}$. is nothing when x is = o & infinite when x is infinited which shews that all the Area between the infinitely produced Asymptote A & the Ordinate MD the infinite in extent is nevertheless finite in Quantity & $2\sqrt{x}$ or $\frac{2x}{\sqrt{x}}$ or 2 xy, or twice the Parallelogram DL. But because the Fluent is infinite when x is so, it shews that all the Asymptotic on the other hand is infinite do wall in Quantity or extent.

Example 6th

To Confirm the foregoing Example, let the same Hy. be refered to the other Asymptote by means of the Ordinate ML. then calling AL x, and ML y, we shall have $y = \frac{1}{xx}$ & $y\dot{x} = \frac{\dot{x}}{xx}$ whose fluent is $= \frac{1}{x}$ Now this fluent when $x = o$, is $= \frac{1}{0}$ or an infinite negative Quantity, when $x = \infty$; the fluent is $= \frac{1}{\infty}$ & when x is infinite the fluent is nothing this shews that all the Area between A & B ML is infinite & is equal to $\frac{1}{x}$ or to the Parallelogram DL and Conf: all the Area Between AC and MD is equal to twice the Parallelogram as before. ———

for a further D, 100 of y^r Example let us imagine another Ordinate Rh to be drawn cutting y^r Asymptote A2 beyond L in h and the fluxes in S, then will the fluent at the Point 2 be $\frac{1}{A2}$ & at y^r Point h $\frac{1}{Ah}$ wherefore the former from the latter that the Acquisition of y^r fluent might be known, in flowing from 2 to h y^r Acquisition will be found to be $\frac{1}{A2} - \frac{1}{Ah}$ and y^r contemporary Acquisition of the Area will be 2 M h But the Acquisition of the Area will be the Contemporary Acquisition of the fluent therefore all the Area between 2 & h will be $\frac{1}{Ag} - \frac{1}{Ah}$ y^t is univerſall whereſoever h exists. Let then h go off to an infinite Distance, & all the Area between 2 & h will be all the infinitely protended Aſymptotic Area, beyond 2 & ſince in this Caſe $\frac{1}{Ah}$ vaniſhes, it follows y^t all the infinitely protended Area beyond 2 will be equall to $\frac{1}{A2}$ & Conſequently will be finite. Let now the Ordinate Rh be taken on ~~the~~ ſide of 2 M. i.-e. let h exiſt between A & 2. Hence ſince the fluent at h is $\frac{1}{Ah}$ & at 2 is $\frac{1}{A2}$ the Acquiſition of the fluent from h to 2 will be $\frac{1}{Ah} - \frac{1}{A2}$ therefore the Area between 2 & h is alſo $\frac{1}{Ah} - \frac{1}{A2}$. Let now h coincide with A & the fluent of the Area between h & 2 will now be the infinitely protended Area between the Aſymptotic A ſ & the Ordinate 2 M & ſince now the Quantity $\frac{1}{Ah} - \frac{1}{A2}$ is infinite; it follows that all the Area between A ſ & 2 M will be infinite.

Let A & B be two Persons beginning the world together, under such Circumstances as in every moment of their Lives afterwards to make equal Increments to their Estates, but let A have the misfortune to be out infinitely in Debt; suppose after a certain time, as T elapsed that the state in Debt should have reduced y infinite Debt to a finite one, viz. we will call it, & that if he was to live on to Eternity the Utmost he could do, would only clear off the Debt, so as at last to be worth but nothing. Whence it is easy to infer, that if his Contemporaneous During the Time T, made an infinite Acquisition, and 2d that if B was to live on to Eternity he could only further acquire the sum Ci. B in the Application it represents y Fluxion B the Area, &c.

Ex: 7th

Let the nature of y Curve AB whose said to its Asymptote AZ, be such that $y = px$ or $y = px \times x^{-n}$ then $\dot{y} = p\dot{x} \times x^{-m}$ whose fluent $\frac{1}{1-m} \times px^{1-m}$, will determine the Asymptotic Space as follows.—

1st If m be less than 1, x^{1-m} will be an Affirmative power of x & therefore in this Case our fluent is $\frac{1}{1-m} \times px^{1-m}$ & will be = 0 when x, ✱ & infinite when x is infinite. Therefore all y Area between A & M P will be finite & equal to y Parallelogram PL multiplied by $\frac{1}{1-m}$ & therefore this Area cat. par. will be greater or less, according as the Quantity $\frac{1}{1-m}$ is >c.

2ly If m be greater than 1. x^{1-m} will be a negative power of x therefore in y^e Case the fluxion $\frac{1}{1-m}x^{1-m}$ will be an infinite negative when $x=0$ & nothing when x is infinite. therefore in this Case the Space betwixt Ev & MP will be infinite and all the space on the other side PA will be finite, & equal to y^e Parallelogram. $\frac{1}{1-m}x^{1-m}$ into the Quantity $\frac{1}{m-1}$ & therefore cat. par. will be greater or less as $\frac{1}{m-1}$ is so.

3ly From the two foregoing Cases, it follows that if $m=1$ as in the Case of y^e common Hyperbola the Asymptotic Spaces on both sides DM will be infinite for the $\frac{1}{1-m}$ or $\frac{1}{m-1}$ will be both equal & cons. both infinite —

*

Example 8.

Tab 5
Fig 1
Page 38

Since in the Common Hyperbola both the Asymptotic Spaces, are infinite, all that can be expected is to measure any particular part of those Spaces —

Thus for Instance Let MM be a common Equilateral Hyperbola, whose Asymptotes are AB & AC, & let it be required to measure the MDPN comprehended between the 2 perpendicular Ordinates MD PN. —

This may be done by various ways; but the most commodious is that which follows.

Bisect DP in B & draw the Ordinate BE Call AB, or, BF, or BP, x. & let the Power of the Hyperbola be 1, then will MP =

$= \frac{1}{r-x}$ & $BQ = \frac{1}{r+x}$. Let the Ordinates MP, BQ. 47
remove into the places np, ng. infinitely near
the former. so that the Space MPQB. may be
open into the Space npgn. then is it plain. that
the Increment or fluxion of this Space will be
$MP\dot{p}n + Bg\dot{g}n = \frac{\dot{x}}{r-x} + \frac{\dot{x}}{r+x}$ now as this fluxion
is not of the simplest form, neither can its
fluent be obtain'd by the simples method.
But to gain this fluent the Quantity must
be thrown into an infinite Series; thus. $\frac{2r\dot{x}}{r^2-x^2} =$
$= \frac{2\dot{x}}{r} + \frac{2\dot{x}x^2}{r^3} + \frac{2\dot{x}x^4}{r^5} + \frac{2\dot{x}x^6}{r^7} + \frac{2\dot{x}x^8}{r^9}$ &c
hence is plain that the fluent of every term
of this series may be found by the simplest method,
& consequently the fluent of the whole will be
$\frac{2x}{r} + \frac{2x^3}{3r^3} + \frac{2x^5}{5r^5} + \frac{2x^7}{7r^7} + \frac{2x^9}{9r^9}$ &c which fluent
will be equal to the Area sought MPQB.
Or thus make $\frac{2x}{r} = A. \frac{Ax^2}{r^2} = B. \frac{Bx^2}{r^2} = C$ &c the
Area MPQB. will be equals to the following
infinite series $\frac{A}{1} + \frac{B}{3} + \frac{C}{5} + \frac{D}{7}$ &c
Cor. whatever be the Quantities r & x, so long
as they have the same proportion one to another
the Area MPQB will be the same
Cor. 2. Or if the Ordinates MP, BQ. be so
taken that AQ shall always have the same
proportion to AP the Area MPQB will be
always the same. For let r & s. be two
given Quantities & let AQ & AP be two
Indeterminate Species. But let AQ be
always to AP :: r : s then will AP + AQ
: AP − AQ :: r + s : r − s. But AQ + AP = 2AB,

C C12 — A2 = 2 B2. & C2 of B. : 2 B2 :: of B. B2 + 8

Therefore of AB : B2 :: + + s : r — s therefore
the Ratio of of B. B2 will always be y^e same
therefore by y^e last Cor. the Area MD2h
will always be the same — —

Cor. 3. If the Common Equilateral Hyperbola
whose Power is unity be referd to its Asymptotes
and if the Difference between any two Abscissæ
be to their sum as 1. : 3 & lastly if we make
$\frac{2}{3} = A \frac{A}{3 3} = B \frac{B}{3 3} = C \frac{C}{3 3} = D \frac{D}{3 3} = E$ then will
the Area belonging to the abovementioned
Abscissæ be $A + \frac{B}{3} + \frac{C}{5} + \frac{D}{7} + \frac{E}{9}$ &c
For in the forgoing computation, the
Difference between x — x & x + x is 2x &
their sum is 2 x, & 2x : 2x :: x : x or :: 1 : 3
therefore $\frac{2x}{x} = \frac{1}{2} 3$ & $\frac{2x}{x}$ or $A = \frac{2}{3}$ ——

Let the Nature of a Curve be $3xx-12x+8=y$ then will its Area be $x^3-6xx+8x$, which Area, wn x is $+$ will be $+$ but it it will also be found that when $x=2$ or 4 the area will be $=0$.

Quere, how must this be conceived? Fig: 3. Tab 3

Since $3xx-12x+8=y$ its evident y^t the curve does not cut its Base (the line upon wch the Absciss's are taken) at y^e beginning of y^e Abscissa, for if x be taken $=0$ y^n will be found $=8$ & not nothing; therefore to find whether this Curve cuts the Base its required to or not & if it does to find in what point, or points it cutts it y must be supposed $=0$ & you will y^e Equation $3xx-12x+8=0$ which being resolved gives $x=2\pm\frac{2}{\sqrt{3}}$ but $\sqrt{3}$ is about $1\frac{3}{4}$ or $\frac{7}{4}$ & therefore $\frac{2}{\sqrt{3}}=\frac{8}{7}$ & $2-\frac{2}{\sqrt{3}}=\frac{6}{7}$ & $2+\frac{2}{\sqrt{3}}=\frac{22}{7}$ nearly. Therefore the Curve cuts the Base in 2 Points whose distances from the Origin of the Abscissa are $\frac{6}{7}$ & $\frac{22}{7}$ nearly. This being understood Let A be the Origin of the Abscissa, & take $AB=\frac{6}{7}$ $AC=\frac{22}{7}$ $AP=2$ $AQ=4$.

1st Therefore it is certain that all the Area standing upon the Base AB is affirmative; but after the Curve has cut its Base in B, the Ordinates will then lye on the other side of the Abscissa so that the so that the Area standing upon BP is negative; now if y^e negative Area be equal & contrary to the affirmative Area standing on the Base AB. its plain that the Area standing upon APB will $=*$. The affirmative and negative Area destroying each other, ——

again, as the Curve has not yet cut its base a second time, the Area upon the Base D will still be negative but after the curve has cut the Base in E the Area will then be affirmative. and if the affirmative Area upon E be equall to the negative one upon D except only in the signe the Area at 2 will ~~be~~ become ~~negative~~ * after which it still grows more & more affirmative in infinitum.

N.B. If you take AB = 2 & thro' D draw DM perpendicular to AB & make MD = 4. then if about the Axis MD with a Parameter = $\frac{1}{3}$ you describe a Parabola, it will be y^e Curve (whose ordinate by perpendicular Ordinate to AE as an Axis) whose nature is here defined — see 2^d Hosp: Art: 308 & seq:

Thus it will be further confirmed if you find the greatest ordinate which will be that whose Abscis is 2 which therefore will be 4. — —

Of the Computation of Brigg's Logarithms.

Theorem.

If the common Hyperbola be referr'd to either of its Asymptotes, by lines parallel to y^e other. I say that all the Asymptotic Spaces, will be Logarithms of the Abscissa, where they terminate provided that those Abscissa be computed from the Center, and all the Asymptotic Spaces from one and the same Ordinate whose Abscis is Unity. Let a & b be the values of 2 Abscissa reckon'd both from the Center Let c & d be y^e Asymptotic Area reaching from the Abscis 1 to y^e Abscis a. & let e be a like area reaching from 1 to b. or from a to ab. (since 1 : b :: a : ab.) then since 1 reaches from 1 to a, & e from a to ab, the Area e & e are such whose addition will always answer to the multiplication of the Abscissa where they terminate. & therefore they are the Logarithms of those Abscissa

To compute the Hyperbolic Logarithm of 10 that is to find in the Equilateral Hyperbola whose power is unity, all the Asymptotic Area from the Abscis 1 to the Abscis 10. —

If $\frac{2}{3}$ be made = A, $\frac{a}{9} = B$, $\frac{b}{9} = C$, $\frac{c}{9} = D$ $\frac{d}{9} = E$ & the Area from the Abscis 1 to y^e Abscis 2 or the Hyperbolic Logarithm of 2 will be $A + \frac{B}{3} + \frac{C}{5} + \frac{D}{7} + \frac{E}{9}$ But the Log: of 8 is 3 times the Log. of 2 therefore

The sum of the series $A + \frac{B}{3} + \frac{C}{3^2} + \frac{D}{3^3}$ &c will be the Area reaching from the Abscis $3-1$ to $3+1$. For by Cor. 3 last Chap. the sum of this series will be the Area Comprehended between the two Ordinates. the Difference of whose abscisses is to the sum of their Abscisses as $1 : 3$ but the Diff. of $3-1$ & $3+1$ is to y^{e} sum of $3-1$ & $3+1$ as 1 to 3 Ergo &c—

Therefore if making $\alpha = \frac{2}{3}$ we make $\alpha = 52$
$= 2$ &c since hence the other Terms, B, C,
D &c. &c as before we shall have $\alpha + \frac{B}{3} + \frac{C}{5}$
$+ \frac{D}{7} + \frac{E}{9}$ &c. the Hyperbolic Log: of 8. —
Again if we make $\frac{2}{9} = b$, $\frac{k}{81} L \alpha \frac{2}{81} = m$
$\frac{m}{81} = n$ &c the Hyperbolic Area ceasing
from 8 to 10 will be $K + \frac{L}{3} + \frac{M}{5} + N$ &c.
But here $\frac{2}{9} = b$ in the foregoing Series therefore
L or $\frac{K}{81}$ or $\frac{B}{81}$ or $\frac{C}{9} = D$. in like manner M
$= B$ N $= K$. &c therefore if to the Hyperbolic
Logarithm of 8 above found &c add B
$+ \frac{D}{3} + \frac{E}{5} + \frac{K}{7}$ &c we shall have the Hyp=
perbolic Logarithm of 10 = 2, 302585092994.
To find the first Term of y° foregoing Series
so, that Bruggi Logarithms may be reduced
by it, i. e. a scale of Logarithms whereon the
Log: of 10 is 1. say as the Hyperbolic
Log. of 10, is to Bruggi Log: of 10 1, 2. as
2, 302585092994 is to 1, so so is 2 the
coefficient of y° first Term $\frac{2}{3}$ of the Series
that produce the Hyperbolic Log: to 0, 868588
9638806 the coefficient of y° first Term of
reduces Bruggi Logarithms, so that if we
make Q; $\frac{0,868588963806}{3} = A \frac{A}{3^2} = B \frac{B}{5^2} = C$ &c.
the sum of the Series $A + \frac{B}{3} + \frac{C}{5}$ will be
Bruggi Logarithm of $\frac{3+1}{3-1}$ for the sum
of this Series will be the Area reaching
from the Abscis 3 − 1 to 3 + 1 or which
is the same thing from the Abscis 1, to

the Absciss whose Value is $\frac{x+1}{x-1}$

Cor. The Hyperbolic Logarithm of any number is to Brigg's Logarithm of the same number as 1, to 434294481903.

The computation of the Hyperbolic Logarithm of 10 to 14 Places, of figures. in order to have 12 of those places exact.

A =	2, 00000000000000	A =	2, 00000000000000
B =	22222222222222	$\frac{B}{3}$ =	7407407407407
C =	24691358024691	$\frac{C}{5}$ =	4938271604938
D =	2743484224497	$\frac{D}{7}$ =	391926321785
E =	304831580055	$\frac{E}{9}$ =	33870175562
F =	3387017562	$\frac{F}{11}$ =	307910687
G =	376335285	$\frac{G}{13}$ =	28948868
H =	41815032	$\frac{H}{15}$ =	2787669
I =	4646115	$\frac{I}{17}$ =	273301
K =	516235	$\frac{K}{19}$ =	27170
L =	57359	$\frac{L}{21}$ =	2731
M =	6373	$\frac{M}{23}$ =	277
N =	708	$\frac{N}{25}$ =	28
O =	79	$\frac{O}{27}$ =	3
P =	9		
Q =	1		

Log: of 8 =	=	2,, 079441541679982	
	B =	22222222222222	
	$\frac{D}{3}$ =	914494741166	
	$\frac{F}{5}$ =	67740351 2	
	$\frac{H}{7}$ =	5913516	
	$\frac{K}{9}$ =	57359	
	$\frac{M}{11}$ =	579	
	$\frac{O}{13}$ =	6	
Log: of 10 = =		2, 302585092994 02	

The Computation of Briggi Logarithm of 2. to 16 Places of Figures, in order to have 14 places exact—

3,/0,8685889638065096

A=	2895296546021679	A=	2895296546021679
B=	-3216996162246315	$\frac{B}{3}$=	1072332054032105
C=	357440180237	$\frac{C}{5}$=	71488806005465
D=	39716002003045	$\frac{D}{7}$=	5671371457186
E=	4412889111145	$\frac{E}{9}$=	490321012388
F=	490321012388	$\frac{F}{11}$=	44574637749
G=	54480112499	$\frac{G}{13}$=	4190777788
H=	6053345833	$\frac{H}{15}$=	403556388
I=	671259398	$\frac{I}{17}$=	39564399
K=	7473266	$\frac{K}{19}$=	3933255
L=	830363	$\frac{L}{21}$=	395411
M=	92262	$\frac{M}{23}$=	40111
N=	10251	$\frac{N}{25}$=	4110
O=	1139	$\frac{O}{27}$=	45
P=	127	$\frac{P}{29}$=	4
Q=	14		
R=	1		

$\log \frac{3+1}{3-1} = \frac{3+1}{3-1} = \frac{4}{2} = 2$

0,3010299956639828

Comp: of Briggi Log: of 3 to 16 Places of Figures

A=	0,0065949625081149	A=	0059496250811149
B=	2081309559	$\frac{B}{3}$=	693768853
C=	802994	$\frac{C}{5}$=	16059
D=	3		

Log: of $\frac{81}{80}$ =	0,0053950318866061
Log: of 80 =	1,9030899869919436
Log: of 81 =	1,9084850188786497
Log of 3 =	477121254719637 4

5.6

The Computation of some of the first Logarithms —
The greatest trouble in computing a Table of Log:
is at the beginning where y^d Series converge slowest.
therefore in such a case it will be proper to look out for
compendiums. thus for brevity's sake we make 86858896 = 2m
then the Log: of 2 being already obtain'd; for y^d Log: of 3
compare y^e Log: of $\frac{3}{2}$ by making $\frac{2m}{5} = A \frac{A}{15} = B \frac{B}{25}$
= &c. having thus got the Log: of $\frac{3}{2}$ add it to y^e Log: of
2 & & you'll have the Log: of 3, or rather thus Log: 2
gives Log: 8. compute y^e Log: of 9 by making $\frac{2m}{17} = A$.
$\frac{A}{289} = B \frac{B}{4913} =$ &c. & the Log: of $\frac{9}{8}$ being thus computed.
and added to y^e Log: of 8 will give y^e Log: of 9, $\frac{1}{2}$ of wch is Log: of 3.
But most compendiously of all thus. y^e Log: of 8 & 10. gives
the Log: of 80, compute y^e the Log: of $\frac{81}{80}$ by making $161 = z$
$\frac{2m}{161} = A \frac{A}{25921} = B \frac{B}{25921} =$ &c. & y^e Log: of $\frac{81}{80}$ being
added to y^e Log of 80 will give the Log: of 81. $\frac{1}{4}$ of wch is
the Log: of 3. The Log^s of 4, 5, 6 are to be had
from those of 2, 10 & 3, so that the Log of 7 comes
next to be computed, which may most compendiously
be done thus. y^e Log^s of 3, 8, 100 give the Log:
of 2400 compute then the Log of $\frac{2401}{2400}$ & add it to
the Log: of 2400 & it will give the Log of 2401, $\frac{1}{4}$ of
which gives the Log: of 7. the Log of 8: 9 may
be had from those of 2 & 3. so that the Log of 11 comes
next to be computed. and now having proceeded so
far we shall have no occasion for any other Artifice, but one, wch will carry us thro all the Prime
numbers which are those whose Log^s only are to
be computed, bec: the Log^s of the rest may
be had from those below y^m Now the nearest
numbers on each side 11 are 10 & 12 whose are
known & consq the Log: of their prod: 120 consq.

the Log of $\frac{121}{120}$ to this added to the Log: of 120 gives the Log: of 121. $\frac{1}{2}$ of which is the Log. of 11 —
in like manner the Log: of 13 may be had by comp.. the Log: of $\frac{169}{168}$ as before be — —

The Demonstration of this is as follows. Viz. in the Application after you have found the Log. of 23 y^e first Term of the series will be abundantly sufficient; even tho' you shd compute all the Logs to 10 Decimal Places in order to have 7 Decimal places just as in the common tables — — —

Demonstration

Let p. be y^e prime number whose Log: is sought all those below it supposed to be known. therefore the Log: of $p-1$ is known. but if p be a prime number, $p+1$ will be an even number, & $\frac{p+1}{2}$ a whole number — whose Log: will therefore be known, but how the Log of $\frac{p+1}{2} \times 2$ that is y^e Log of $p+1$ is known. but $\frac{p+1}{2} \times p-1 = pp-1$ therefore the Log of $pp-1$ is known, compute the Log: of $\frac{pp}{pp-1}$ & this added to the Log: of $pp-1$ will give the Log: of pp. & con: of p. Q E D —

59

The manner of finding the Compendium is this.

To compute the Logarithm of any number as p, is to compute the Idea from the Abscisse 1 to the Abscisse p. & by Cor: 3 of the last Chap. z is found by saying $z-1 : z+1 :: 1 : z = \frac{p+1}{p-1}$

Now y^e Compendium consists in having z an Integer because not only the trouble of fractions is avoided, but Likewise because the greater z is, the faster will the series converge, which when $z = \frac{p+1}{p-1}$ would be exceeding slow.

Wherefore if instead of computing the Log of p, you compute the Log of $\frac{p}{p-1}$ or of $\frac{p^n}{p^{n-1}}$ z will be an Integral number $= 2p-1$, or $2p^n-1$, & you will have the Log of p^n & only by n times the Log: of p. Examine therefore the Powers of p. & if there be any power of p such as the n^{th} Power, of y^e the Log: of p^n-1, can be had from those y^t are already known, i.e. if all the Logs of all the component prumes of p^n-1 are known. y^t power of p will afford the compendium. ———

Thus if $p = 5$ $p^2 = 49$ $p^{2-1} = 48 = 2 \times 2 \times 2 \times 2 \times 3$, or $p^4 = 2401$. & $p^{4-1} = 2400 = 3 \times 8 \times 100$, the Logs of which numbers are known, wherefore $\frac{49}{48}$ or $\frac{2401}{2400}$ are the fractions sought. —

N.B. The greater n is the faster will the series converge But after you have computed a few of y^e first Logs: if $n = 2$ the series will converge fast enough, & if p be a prime number p^2 will always be the power of p sought. for $pp-1 = p+1 \times p-1$, the Logs of both which numbers may be had, from those below, as was before demonstrated—

To Demonstrate that the increasing or decreasing of the Logarithms in any System proportionally, does not alter their Properties as Logarithms ——

Let a & b, be 2 numbers whose Logarithms in some System, viz: Brigg's are p & q, then will p+q be the Log: of their Prod: ab, magnr now every Log: in this System to be chang'd in the Prop: of 1 to m. then will the Log of a become mp that of b, qn, & of their Prod: mp+qn.: but mp+qn is as much the sum of mp & nq; as p+q was of p & q before. Therefore the Log! thus altered will still retain their fundamental Property, viz. they had before, (viz) that their Additions answers to the Multiplication of their natural numbers to which they belong: & therefore all properties deriv'd from those fundamentals will still be the same ——

Cor. 1 Hence it follows, viz. in any two Systems, whatever, y^e Log^s of y^e same Numb^rs are proportionable ——

Cor. 2. By substituting proper numbers for m, by means of Brigg's System only, may be had all the other Systems of Log! thus if m = 2, 3025 &c the Hyperbolic Log: will be had viz. it is a System wherein the Log of 10 is to 2 :: 3025 &c ——

If it were required to find a System wherein the Log: of 2 should be 1, 000 &c put m = 1 &c / 0,3010299 = 3, 3219 &c. by which have the System sought ——

6/

Tab. 2. Fig. 8. Page 49.

Prob. 7th

To find the Content of a Solid, generated by the revolution of any known Curve AM about its Axis AP. — Let the Diameter of a Circle to the Ordinate PM, & let the Ordinate PM make an infinitely small revolution into the Plan of the Parallelogram. This represents the fluxion of the Area APM, & Cylinder generated by its Revolution about AP will represent the fluxion of a solid generated by the whole APM. But the Basis of such such a Cylinder is $\frac{1}{4}\pi y^2$ & its height is $\dot{x}$; compute therefore the fluent of $\frac{1}{4}\pi y^2 \dot{x}$ & that fluent will determine the Solid, in the same manner as the fluent of $y\dot{x}$ determined the Area. ——

Ex 1. Let the nature of the Curve by whose Area the solid is generated be expressed by this Equation $px^m = y^n$ supposing m & n integral and affirmative then will $y = px^{\frac{m}{n}} = p^{\frac{1}{n}} \times x^{\frac{m}{n}}$

$\frac{1}{4}\pi yy\dot{x}$

$\frac{1}{4}\pi yy = p^{\frac{2}{n}} x^{\frac{2m}{n}}$ $\frac{1}{4}\pi yy\dot{x} = \frac{1}{4}\pi p^{\frac{2}{n}} \times \dot{x} \times x^{\frac{2m}{n}}$

the fluent whereof is $\frac{n}{2m+n} \times \frac{1}{4}\pi p^{\frac{2}{n}} \times x^{\frac{2m}{n}+1} =$

$\frac{n}{2m+n} \times \frac{1}{4}\pi yyx$. But $\frac{1}{4}\pi yyx$ is a Cylinder having the same Base and Altitude with the Solid propos'd. Therefore the Content of the Solid propos'd is equal to a Cylinder of the same Base & Altitude multiply'd into the fraction $\frac{n}{2m+n}$ —

Ex: 2. Let it be required to find the Content of the Solid or rather part of y^2 Solid generated by the Revolution of ye curve DH about one of its Asymptotes AS. Let the nature of the Curve when referred to its Asymptote AS be $y = \frac{p}{x^n} = p x x^{-n}$ then will $yy = p^2 x x^{-2n} \dot{x} = 2p^2 x^{-2n} \dot{x}$ whose fluent is $\frac{1}{1-2n} \times 2p^2 x^{1-2n}$ which shows yt. if 2n be less than 1. all the Solid generated by the Area between ADHM is finite & equal to the Cylinder generated by the Parallelogram DL multiplied into the fraction $\frac{1}{1-2n}$. the other part of y^2 Solid being infinite. On the other Hand if 2n be greater than 1. all the Solid between ADHM will be infinite & the infinitely extended Solid on the other side MD will be finite & equal to the abovementioned Cylinder multiplied into the fraction $\frac{1}{2n-1}$ of this latter kind among many others. is the Solid generated by the Hyperbola which will always be equal to the Cylinder generated by the Parallelogram DL.

Lit 3. To find the Content of a Spheroid generated by the Revolution of a semi Ellipse AHBa about its Axis Aa. Call CA t, CB x, CB c, HD y & we shall have from the nature of ye Ellipse $yy = cc - \frac{ccxx}{tt}$ whose fluent $ccx - \frac{cccx^3}{3tt}$ is the content of the Solid generated, by the Area CBHD. & therefore if $x = CA$ or t & we shall have the Hemispheroid generated by the Area CBA $= cct - \frac{cct^3}{3tt} = \frac{2}{3} ccc \times t$ therefore the Content of the whole Spheroid $= \frac{2}{3} ccc \times 2t$. But $ccc \times 2t$

Tab. 3. Page. 64

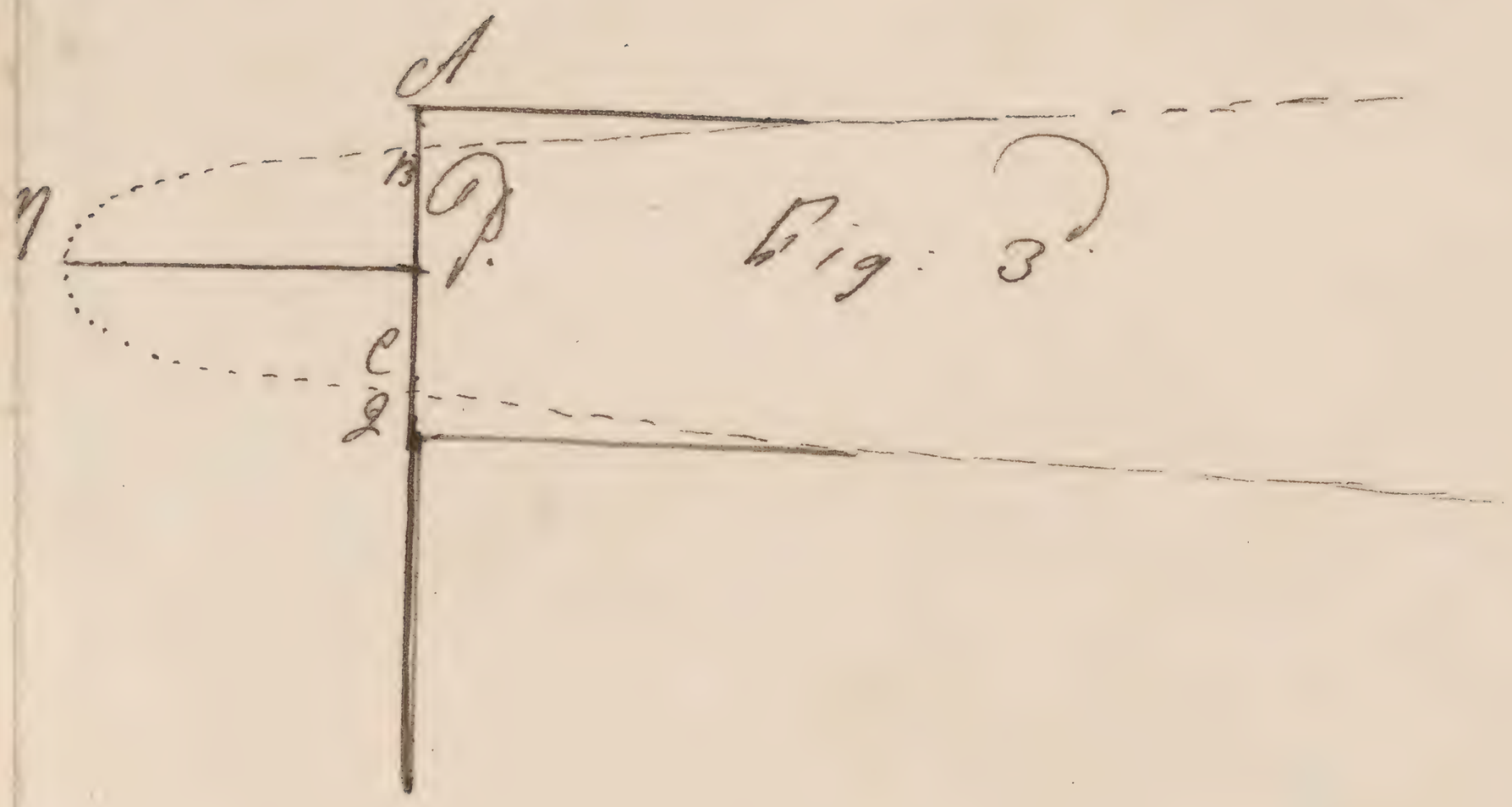


Fig. 3.

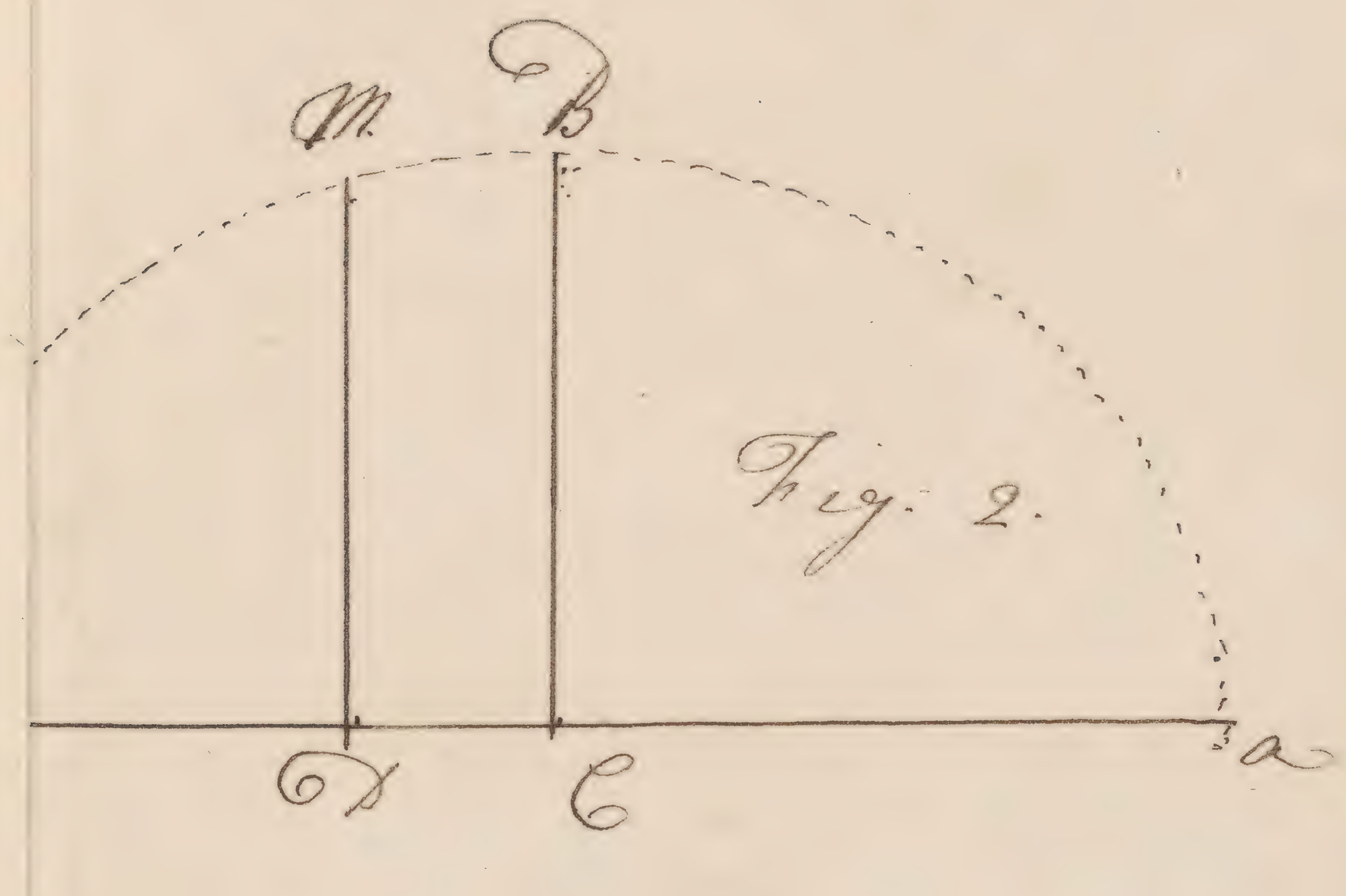


Fig. 2.

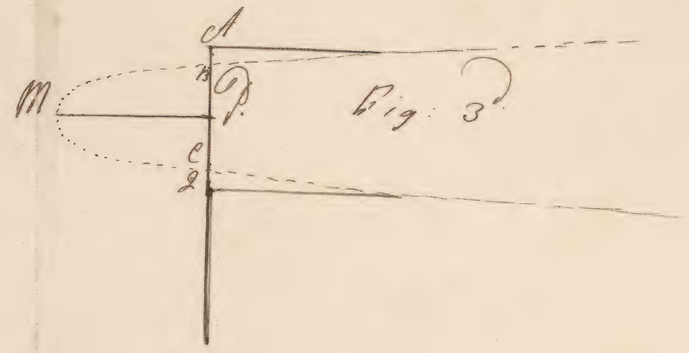
A

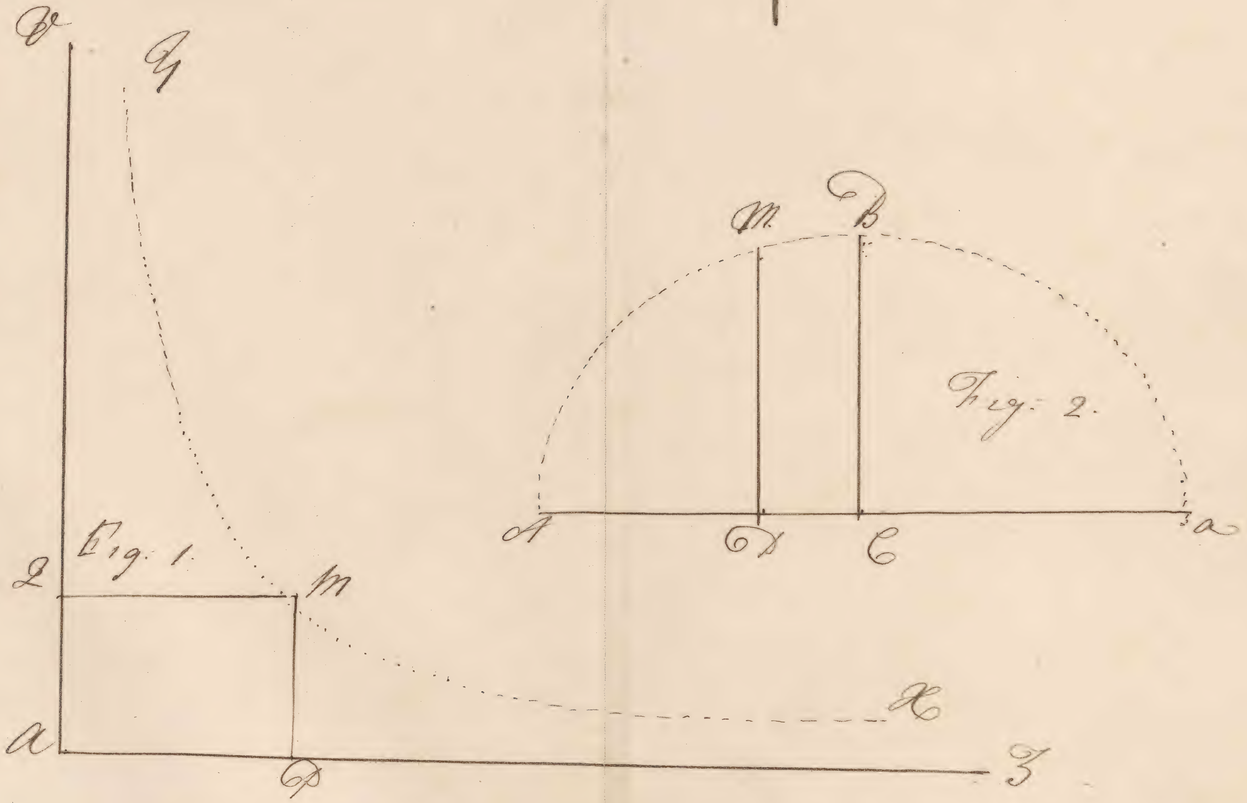
3

is the Content of a circumscribing Cylinder. 64

* & $\dot{e}\dot{y}\dot{y}\dot{x} = ecc\dot{x} - \frac{ecc\dot{x}\dot{x}\dot{x}}{tt}$ whose fluent &c.

Tab: 3. Page: 64





Prob. 15

Fab. 2.

Fig. 3

Page 49.

As the Curvilinear Area AMB revolving about its Axis AB, generates a Solid: so the Curve, am by such a revolution generates the surface of the Solid. Let it therefore be required to find the surface generated by any known Curve AM revolving about an Axis AP. & let the Diameter of the Circle be to the Circumference as 1 to π & take AP = x, PM = y, then will AM. or $\sqrt{MP^2+PM^2} = \sqrt{x^2+y^2}$. But as PM is the fluxion of the Curve AM. if AM by its Revolution generates a surface. PM will generate the fluxion of that surface, which will be an Annulus whose Breadth is PM. & whose Circumference is equals to 2π Circumference of a Circle described by the Semidiameter PM. therefore $2\pi y \times \sqrt{\dot{x}^2+\dot{y}^2}$ will be the fluxion of its surface, and its fluent when found from the Nature of the Curve will determine the surface sought.

$\times \delta y \times \sqrt{x'^2+y'^2} = r\theta \delta\theta \delta y \times \sqrt{\dot{x}^2+\dot{y}^2}$ the fluxion
of the surface sought $= e \times r\theta \delta\theta$ —

Let AMa be a semicircle upon the Center C
& let it be required to find the Surface of the Shell
generated by its Revolution about it's Diameter.
Ax. Call AC r. AD x DM y. Da 2r − x &
shall have yy = 2rx − xx & 2yẏ = 2rẋ − 2xẋ. ẏ =
$\frac{r\dot{x} - x\dot{x}}{y}$. & $\dot{y}\dot{y} = \frac{r^2\dot{x}^2 - 2rx\dot{x}^2 + x\dot{x}\dot{x}}{yy} = \frac{rr\dot{x}^2 - yy\dot{x}^2}{yy} =$
$= \frac{rr\dot{x}^2}{yy} - \dot{x}\dot{x}$. Therefore $\dot{x}\dot{x} + \dot{y}\dot{y} = \frac{rr\dot{x}^2}{yy}$. Ergo
$yy \times \dot{x}^2 + \dot{y}^2 = rr \times \dot{x}^2$ & $y \times \sqrt{\dot{x}^2 + \dot{y}^2}$ * (the fluxion
of the Area sought) $= c \times 2r\dot{x}$ whose fluxent
$c \times 2rx$ will be the Surface of a Segment of a Shell
whose Axis is AD. x. But from the similar
Triangles DMC, AMa we have. AD. AM :: AM. aa
& Conſt. AD × aa = AM² or 2rx = AM². Therefore
$c \times 2rx = c \times AM^2$ but $c \times AM^2$ is y^d Area
of a Circle whose Radius is AM. Therefore
the Surface of any Segment of a Shell is equal
to a Circle, whose semidiameter is a Line
drawn to the Circumference of it's Base
from the Pole of y^e Segment. Therefore the
Surface of the whole Shell is equal to a
Circle whose Radius is the Diameter of
the Shell a which is the same it is
equal to 4 great Circles of the Shell

Ex 2. Let it be required to ascertain the surface of 4th Parabolic Solid, generated by a common Parabola about a Diameter. Let Y be the Focus of the Parabola, & call AP, x PM, y. & we shall have $y\dot{y} = 4mx$ & $2y\dot{y} = 4m\dot{x}$ & $y\dot{y} = 2m\dot{x}$ & $yy\dot{y}^2 = 4m^2\dot{x}^2$ or $4mx + 4m^2 \times \dot{x}^2$ Therefore $y \times \sqrt{\dot{x}^2 + \dot{y}^2} = * y \times \sqrt{\dot{x}^2 + \dot{y}^2} = 2e\dot{x} \times \sqrt{mx + mm}$ now to reduce this fluxion $2e\dot{x} \times \sqrt{4mx + 4mm}$. into a more convenient form, for the better finding of its fluent, make $x + m = z$ Then we shall have $\dot{x} = \dot{z}$ & $4mx + 4mm = 4mz$ & $2e\dot{x}\sqrt{4mx + 4mm} = 2e\dot{z} \times \sqrt{4mz}$ whose fluent is $2e \times \frac{2}{3} z \times \sqrt{4mz}$ But at the Beginning of the surface of 4th Parabolic Sol. that is when $x = 0$ this fluent is $2e \times \frac{2}{3} m \times \sqrt{4mm}$. which last being subtracted from the foregoing fluent, to render it equal to the surface sought, we shall have 4t. surface $= 2e \times \frac{2}{3} z \times \sqrt{4mz} - 2e \times \frac{2}{3} m \times \sqrt{4mm}$. Now to construct this fluent that is to find some Geometrical Area with which it may be compared.

Set off on the Point P on the Axis contrary to PA the Distance $PL = eth$. & the' PL & L draw the Chord LQ perpendicular to the Axis & since $PL = AL$ we shall have $AQ = AP + AP = x + m$ $= z$ & $PQ = \sqrt{4mz}$ & $\frac{2}{3} z \times \sqrt{4mz} = \frac{2}{3} AQ \times PQ =$ to the Parabolic Space AQP,

therefore $2 \times \frac{2}{3} z \times \sqrt{4mz} = 2 \times \frac{2}{3} m \times \sqrt{4mm}$ = the Area ALL & $2 \times \frac{2}{3} z \times \sqrt{4mz}$ —

$= 2 \times \frac{2}{3} m \times \sqrt{4mm} =$ the Area of Mnb & $2ex \times$
$\times \frac{2}{3} \times \sqrt{4m^3} - 22 \times \frac{2}{3} m \times \sqrt{4mm} =$ the afores'd
Area multiply'd into e. Therefore as the
Diameter of a Circle is to its Circumference
so is the Area of Mnb to the Surface sought.

Jab. 4
Fig 2
Page 73

To find the Circumference of a Circle —

Prob. 9th —

Tis required, having given the Radius of any Circle together with BC the Tangent at D. to find that Arc & Conf. from the given Tangent of some known Arc to find the whole Circumference.

Draw the Diam. ADC & imagine another, as Ac infinitely near it drawn & perpendicular to AC; then from the similar Triangles Cc & ABC as also ACc & & ADC we have the following proportion Cc : Bb :: AC or ac. : AB or :: ABx AB x or AC : Cc : BD :: AC : AD or :: AC : AB or :: AB x AC : AB². therefore ex æqua Cc : BD :: AC² : AB².

Now to render this Calculation more simple, Call AB 1, BC t then will Cc = t² AC² = 1 + t² BD = $\frac{t^2}{1+t^2}$ = t² − t⁴ + t⁴t² − t⁶t² + t⁸t² &c when BD = $t - \frac{t^3}{3} + \frac{t^5}{5} + \frac{t^7}{7} + \frac{t^9}{9}$ &c —

Now to find the whole circumference of BD be some known Arc, whose tangent is given, thus. Let BD be an Arc of 45°. then will its tangent BC = Rad or 1, & we shall have BD = $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$ &c & Conf. 8 BD, or the whole Circumference will be 8 times the foregoing Series. —

But as this Series however simple it may be converges to slow for practice, it will be proper to take a less Arc than that of 45°. Whose Tangent can be computed. As for Instance let BD be an Arc of 30° & produce its Tangent BC to F till BF = BC then drawing AF tis evident that ACF will be an Equilateral Triangle & Conf. that AC = $\frac{1}{2}$ CB or 2t

Whence $AC^2 - CB^2 (4tt - tt) = AB^2$ &c. conf. tt
$= \frac{1}{3}$ Whence $t = \frac{1}{\sqrt{3}}$ $t^3 = \frac{1}{\sqrt{3} \times 3}$ $t^5 = \frac{1}{\sqrt{3} \times 9}$ $t^7 = \frac{1}{\sqrt{3} \times 27}$
$t^9 = \frac{1}{\sqrt{3} \times 81}$ &c Therefore in this Case $BD = \frac{1}{\sqrt{3}} -$
$- \frac{1}{\sqrt{3} \times 3 \times 3} + \frac{1}{\sqrt{3} \times 9 \times 5} - \frac{1}{\sqrt{3} \times 27 \times 7} + \frac{1}{\sqrt{3} \times 81 \times 9}$ &c. $=$
$= \frac{1}{\sqrt{3}} \times 1 - \frac{1}{3 \times 3} + \frac{1}{9 \times 5} - \frac{1}{27 \times 7} + \frac{1}{81 \times 9}$ &c. ——————

Therefore $6BD = \frac{6}{\sqrt{3}} \times$ the foregoing Series.
But $6BD$ is the Circumference of a Circle whose Diameter is 2 or a whole Circumference whose Diameter is 1 &c $\frac{6}{\sqrt{3}} = \sqrt{12}$ for if we make $x = \frac{6}{\sqrt{3}}$ we shall have $xx = \frac{36}{3} = 12$ &c $x = \sqrt{12}$.
Therefore the Circumference of a Circle whose Diameter is $1 = \sqrt{12} \times 1 - \frac{1}{3 \times 3} + \frac{1}{9 \times 5} - \frac{1}{27 \times 7} + \frac{1}{81 \times 9}$ &c.
or if we make $\sqrt{12} = A$ $\frac{A}{3} = B$ $\frac{B}{3} = C$ $\frac{C}{3} = D$ &c.
then we shall have the Circumference of a Circle whose Diameter is $1 = A - \frac{1}{3}B + \frac{1}{5}C - \frac{1}{7}D + \frac{1}{9}$ &c. $+$ ——
See the work in Sherwins Tables, Diameter being 1, the Circumference $= 3,14159\ 26535\ 9$.

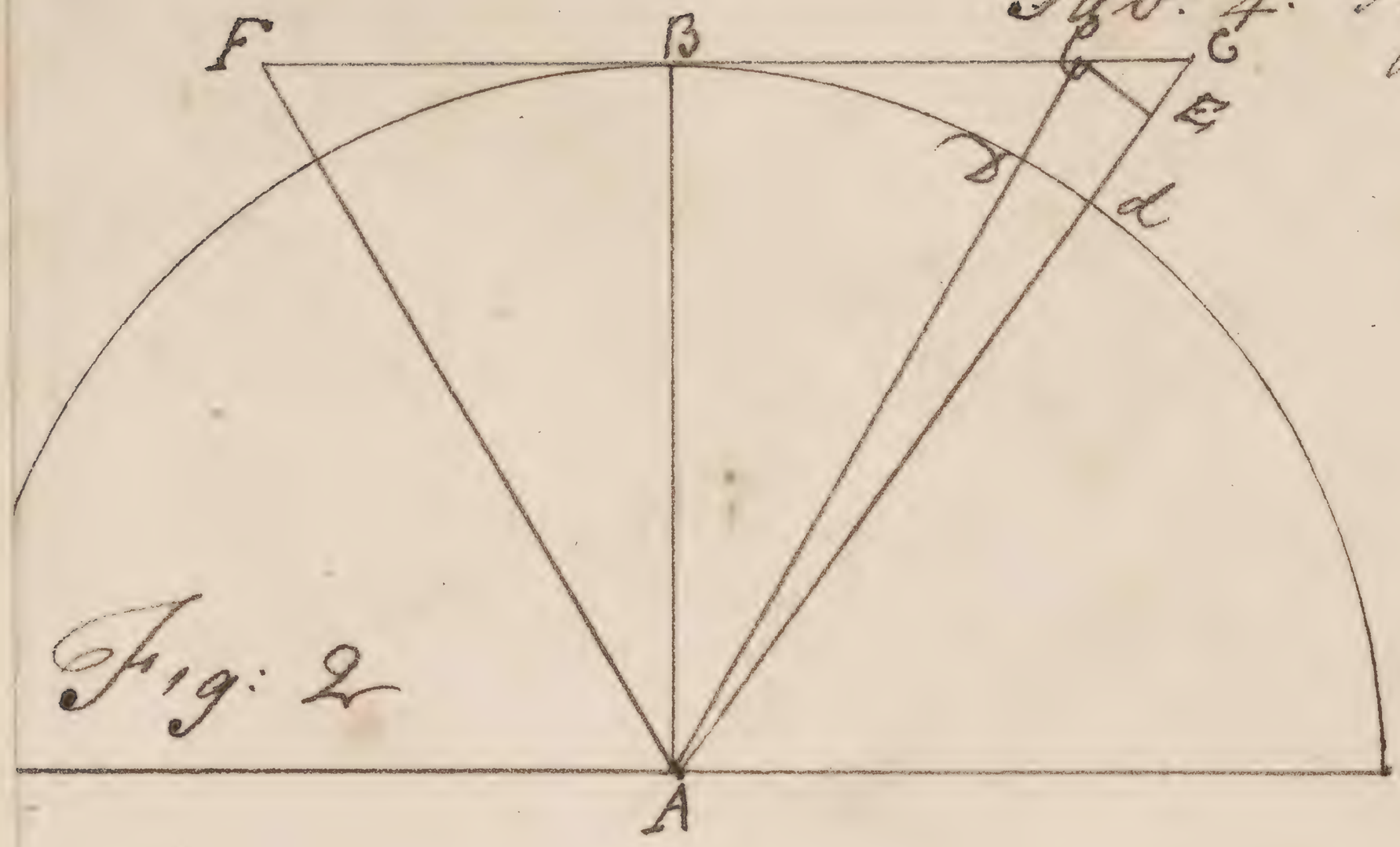


Fig: 2

Tab: 4: Page — 93.

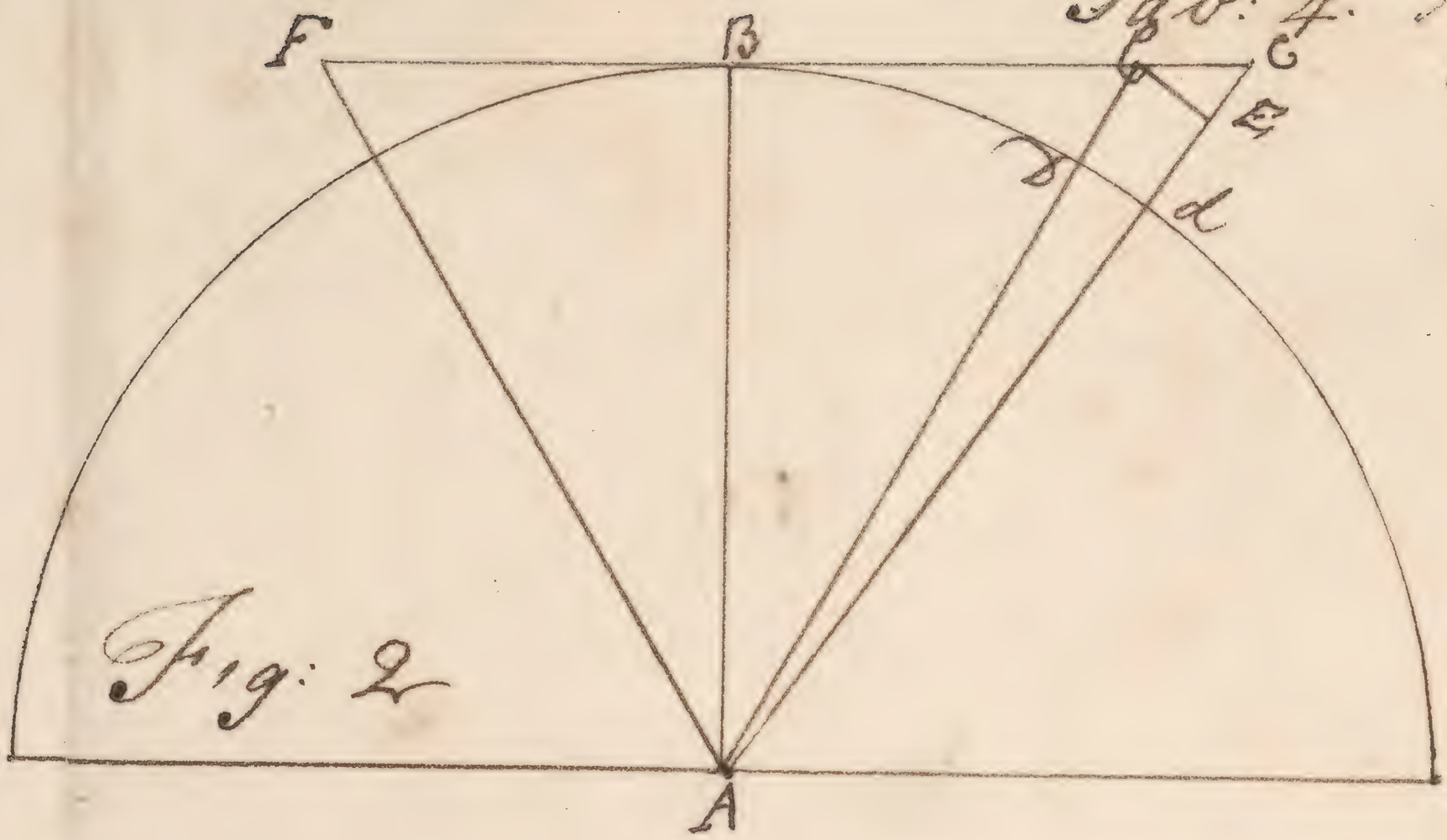
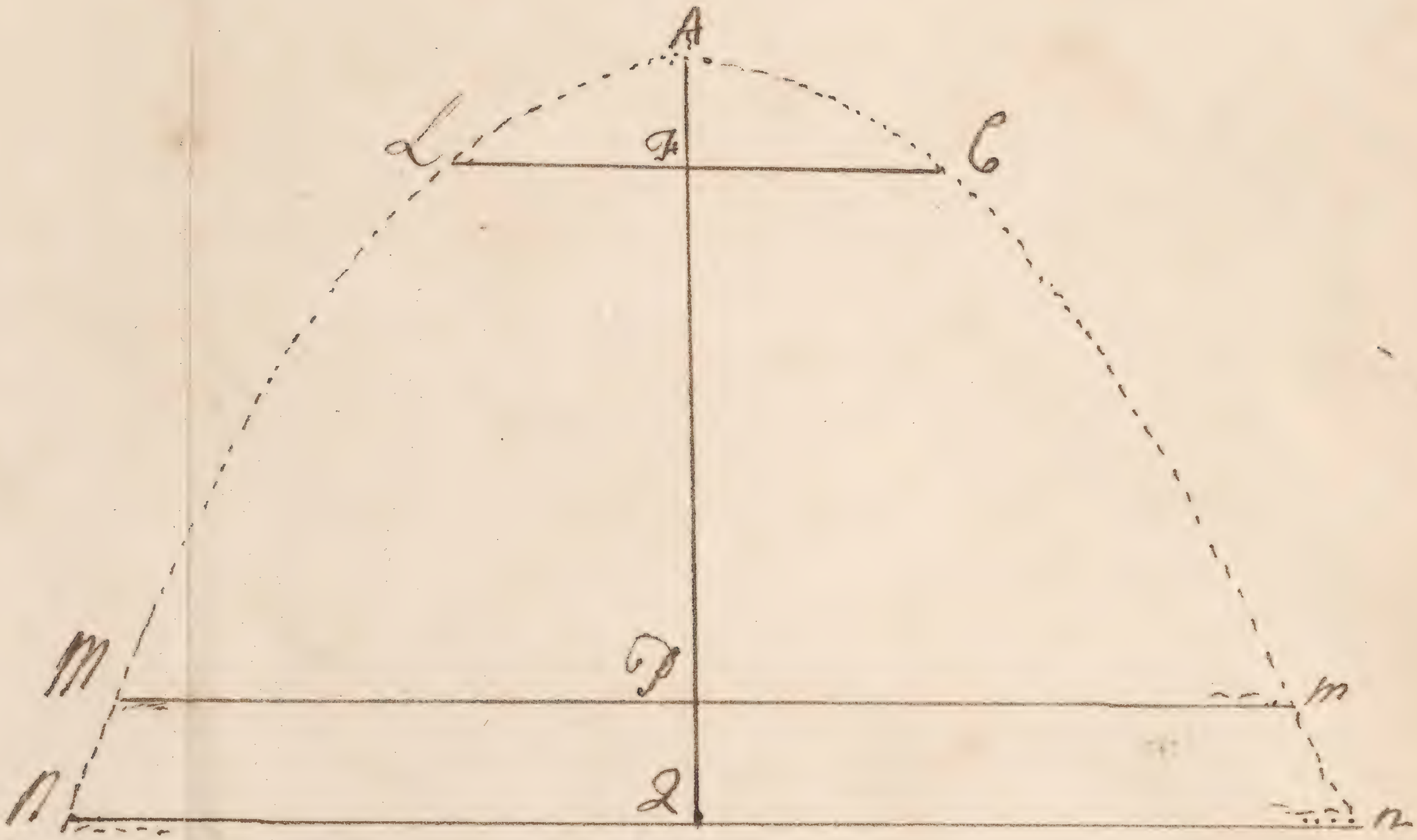


Fig: 2



The method of Reverting Series

(Tab 5 Fig 1 Page 85.)

This will be best seen by y^e following Examples.

Ex: 1. Let MN be the common Hyperbola and to its Asymptote AB. Let BC be a fix'd Ordinate & BA a moveable one 'tis required, having given the Base BZ to find the Area & v: v: having given the Area BCNZ. to find the Base or alone BZ.

Solution

Let x be the Power of the Hyperbola call'd also AB.1, BZ.$\dot{x}$, & NZ.$\dot{y}$, & the Area BCNZ.$\dot{z}$. Then we shall have $y = \frac{1}{1+x}$ & $y\dot{x}$ or $\dot{z} = \frac{\dot{x}}{1+x} = \dot{x} - x\dot{x} + xx\dot{x} - x^3\dot{x} + x^4\dot{x}$ &c. $z = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5 - \frac{1}{6}x^6$ &c. & thus the first part of the problem is dis=patch'd. Let it now be required having given $\dot{z}$ or the Area* to find x or the Base BZ which is as much as to say, that as we have already express'd the Value of $\dot{z}$, by an infinite series, wherein the several powers of x are concern'd; 'tis now required to express the value of x, by an Infinite Series wherein the several Powers of $\dot{z}$ are concern'd. In order to which its necessary that the several powers of $\dot{z}$ should be first found to as many terms as the series is intended to be continued — Thus

*NN2

From the solution of y^e Problem it appears, that if there be any Equilateral Hyperbola whose power is 1, & z be any Asymptotic Area, comprehended between y^e Ordinate 1, & the Ordinate 1 — thou invtd x be the Basis of that Area, and $\overset{+x}{)}$ its value will be found by the following infinite Series, viz^t

$$x = z + \frac{az^2}{2} + \frac{Bz^3}{3} + \frac{Cz^4}{4} + \frac{Dz^5}{5} + \frac{Ez^6}{6} \&$$

where A, B, C, D represent the several Terms as they come out ——

S^r Isaac's Theorem was found, by, receiving the Series $ax + bx^2 + cx^3 \&$ in the manner above shewn ——

Equation 1st. $\zeta = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \frac{1}{5}x^5$ } 6

Equation 2. $\zeta^2 = * + x^2 - x^3 + \frac{11}{12}x^4 - \frac{5}{6}x^5$.

Equation 3. $\zeta^3 = * \quad * + x^3 - \frac{2}{3}x^4 + \frac{7}{4}x^5$

Equation 4th. $\zeta^4 = * \quad * \quad * + x^4 - 2x^5$.

Equation 5th. $\zeta^5 = * \quad * \quad * \quad * + x^5$.

Having got these Equations, the whole Business of this Section, is from these Equations to find out, out of which all the Powers of x except x^1 first shall be expung'd successively, so that at last x may be found equall to an infinite series compos'd of the ascending powers of ζ. As for Instance Equation 1st $x - \frac{1}{2}x^2 + \frac{1}{3}x^3 \&c = \zeta$ Now the Term $-\frac{1}{2}x^2$ is to be expung'd by that of the 2d Equation thus. $x^2 - x^3 + \frac{11}{12}x^4 - \frac{5}{6}x^5 = \zeta\zeta$ therefore $\frac{1}{2}x^2 - \frac{1}{2}x^3 + \frac{11}{24}x^4 - \frac{5}{12}x^5 = \frac{1}{2}\zeta\zeta$; which last Equation added to the first will give Equation 6th $= x * - \frac{1}{6}x^3 + \frac{5}{24}x^4 - \frac{12}{60}x^5 = \zeta + \frac{1}{2}\zeta^2$. Now it's easy to see that the next Term to be expung'd is $\frac{1}{6}x^3$ which is done by Equation 3d thus. $x^3 - \frac{2}{3}x^4 + \frac{7}{4}x^5 = \zeta^3$ Therefore $\frac{1}{6}x^3 - \frac{1}{9}x^4 + \frac{7}{24}x^5 = *\frac{*}{6} + \frac{1}{6}\zeta^3$ this last Equation added to the 6th gives Equa: 7 $= x * * - \frac{1}{24}x^4 + \frac{3}{40}x^5 = \zeta + \frac{1}{2}\zeta\zeta + \frac{1}{6}\zeta^3$. The next Term $-\frac{1}{24}x^4$ is expung'd by Eq: 4th thus $x^4 - 2x^5 = \zeta^4$, therefore $\frac{1}{24}x^4 - \frac{1}{12}x^5 = \frac{1}{24}x^4$ This added to Eq: 7th will give Eq: 8 $= x * * * * - \frac{1}{120}x^5 = \zeta + \frac{1}{2}\zeta^2 + \frac{1}{6}\zeta^3 + \frac{1}{24}\zeta^4$ The last Term to be expung'd is $\frac{1}{120}x^5$ which is effected by the last Eq: 5th thus —

Sr Isaac Newtons Theorem for reverting Series of y^2 aforegoing Lemd. ins wh the Quantity is ran into in every power ——

Sit. $z = ax + bx^2 + cx^3 + dx^4 + ex^5$, &c vanishing ext.
$x = \frac{z}{a} - \frac{b}{a^3} z^2 + \frac{2b^2 - ac}{a^5} z^3 + \frac{5abc - 5b^3 - a^2 d}{a^7} z^4 +$
$+ \frac{3a^2c^2 - 21ab^2c + 6a^2bd + 14b^4 - a^3e}{a^9} z^5$

From the Solution of the first part of y^e Example may be drawn the following useful Observations.

(1.) That the fluent of this fluxion $\frac{x\dot{x}}{z}$ is y^e Hyperbolic Logarithm of z. For instead of $\frac{BC}{AB}$, we call $\frac{x}{z}$ the fluxion of the Area BCN and will be $\frac{\dot{x}}{z}$ Therefore the fluent of $\frac{\dot{x}}{z}$ is the Area BCN which (supposing AB = 1) is the Hyperbolic Log. of the Abscis AL as above ——

Exp. 2. This required having given AB the Radius of any Circle to find the Line sin. Arc as BC & sin. that Arc = v. v: ——

(Tab. 4 Fig: 2 Page. 73)

Let CD be an Ordinate Infinitely near & parallel to ED, & Let CE be parallel to AB. call AB, 1 ED, y. BC z. CE ẏ & CA $\sqrt{1-yy}$ Then from the similar Triangles ADC, CEC, we shall have this proportion. $AD\sqrt{1-yy} : AC(1) :: CE(\dot{y}) :$
$CC(\dot{z}) = \frac{\dot{y}}{\sqrt{1-yy}} = \dot{y} \times \frac{1}{2} y^2 \dot{y} + \frac{3}{8} y^4 \dot{y} + \frac{5}{16} y^6 \dot{y} +$ &c.
Therefore $z = y + \frac{1}{6} y^3 + \frac{3}{40} y^5 + \frac{5}{112} y^7$ &c which is the first part of the Problem ——

The other Part consists in computing the following Equations —— ——

From this Observation that the fluent of $\frac{\dot{x}}{x}$ is the Hyperbolic Log. of x it will appear that the fluent of $\frac{\dot{x}}{x}$ is the Hyperbolic Log. of $\frac{1}{x}$ For $-\frac{\dot{x}}{x} = -1 \times + \frac{\dot{x}}{x}$ but the fluent of $+\frac{\dot{x}}{x}$ is the Log. of x therefore the fluent of $-1 \times \frac{\dot{x}}{x} = 1 \times \text{Log. } x$ $= \text{Log. of } \frac{1}{x}$ Therefore the fluent of $-\frac{\dot{x}}{x}$ is the Hyperbolic Log. of $\frac{1}{x}$.

$1 \ldots z = y + \frac{1}{6}y^3 + \frac{3}{4}y^4 + \frac{5}{112}y^5$

$2 \ldots z^2 = * \pm y^3 + \frac{1}{2}y^5 + \frac{37}{120}y^7$

$3 \ldots z^3 = * * + y^5 + \frac{5}{6}y^7$

$4 \ldots z^4 = * * * * + y^7$

$5 \ldots z - \frac{1}{6}z^3 = y * = \frac{1}{120}y^5 \frac{34}{5040}y^7$

$6 \ldots z - \frac{1}{6}z^3 + \frac{1}{120}z^5 = y * \pm \frac{1}{5040}y^7$

$7 \ldots z - \frac{1}{6}z^3 + \frac{1}{120}z^5 \frac{1}{5040}z^7 \&c = y$ or thus

$$y = \frac{z}{1} - \frac{z^3}{1\times2\times3} + \frac{z^5}{1\times2\times3\times4\times5} - \frac{z^7}{1\times2\times3\times4\times5\times6\times7}$$

Sr Isaac Newtons Theorem for resolving Series of this kind, (when z) when the Quantity is concealed, but in every other power beginning with the first Power

Let $z = ay + by^3 + cy^5 + dy^7 + ey^9$ &c. then is

$$y = \frac{z}{a} - \frac{b}{a^4}z^3 + \frac{3b^2 - ac}{a^7}z^5 + \frac{8abc - a^2d - 12b^3}{a^{10}}z^7 + \frac{55b^4 - 55ab^2c + 10a^2bd + 5a^2c^2 - a^3e}{a^{13}}z^9 \&c$$

N.B. The square Root of the Series $1 - \xi^2 + \frac{1}{3}\xi^4 - \frac{2}{45}\xi^6 \pm \frac{1}{345}\xi^8$ must be got by the common method of extracting y^e Sq. Root

Newtons Theorem is for y^e Binomial Limes only, & therefore can't be apply'd here.

From the Solution of these Problems may be got y^e following Series for finding the Sine or Cosine of any proposed Arc, viz. if ξ be any Arc whose Sine is y we shall have

$$y = \xi - \frac{A\xi^2}{2\times 3} - \frac{B\xi^2}{4\times 5} - \frac{C\xi^2}{6\times 7} - \frac{D\xi^2}{8\times 9} \text{ &c}$$

But if y be the Cosine we shall have

$$y = 1 - \frac{A\xi^2}{1\times 2} - \frac{B\xi^2}{3\times 4} - \frac{C\xi^2}{5\times 6} - \frac{D\xi^2}{7\times 8} \text{ &c, &c}$$

from Radius or 1 we take the Cosine, we shall have the Versed Sine of the Arc ξ

$$= \frac{A\xi^2}{1\times 2} + \frac{B\xi^2}{3\times 4} + \frac{C\xi^2}{5\times 6} + \frac{D\xi^2}{7\times 8} \text{ &c, A being} = 1.$$

Problem.

It is required to find by an Infinite Series, the Cosine of any proposed Arc, in a Circle whose Radius is 1.

Let the Arc proposed be z & its sine y, then will its Cosine be $\sqrt{1-yy}$, but by the last problem $y = z - \frac{1}{6}z^3 + \frac{1}{120}z^5 - \frac{1}{5040}z^7$ &c. Therefore $yy = z^2 - \frac{1}{3}z^4 + \frac{2}{45}z^6 - \frac{1}{315}z^8$ &c. Therefore $1-yy = 1 - zz + \frac{1}{3}z^4 - \frac{2}{45}z^6 + \frac{1}{315}z^8$ &c. Therefore $\sqrt{1-yy}$ or the Cosine sought $=$

$$1 - \frac{1}{2}z^2 + \frac{1}{24}z^4 - \frac{1}{720}z^6 + \frac{1}{40320}z^8 \text{ &c} = 1 - \frac{z^2}{1\times 2} + \frac{z^4}{1\times 2\times 3\times 4} - \frac{z^6}{1\times 2\times 3\times 4\times 5\times 6} + \frac{z^8}{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8} \text{ &c}$$

Scholium.

From the solution of this Problem but one it appears, if y be a right line & z an Arc of a Circle whose Radius is r, the Fluxion of the Arc will be $\frac{r\dot{y}}{\sqrt{rr-yy}}$ by viz: The fluxion of $\frac{r\dot{y}}{\sqrt{rr-yy}}$ is an Arc whose right sine is y & of a Circle whose Radius is r. & from hence again it follows, that if x be the versed sine of an Arc of a Circle whose Radius is 1, the fluxion of the Arc will be $\frac{r\dot{x}}{\sqrt{2rx-xx}}$ by viz: For in this Case $2rx - xx = yy$. Therefore $2r\dot{x} - 2x\dot{x} = 2y\dot{y}$. Therefore $\frac{r\dot{x}}{y} = \frac{r\dot{y}}{r-y}$. Substitute $\sqrt{2rx-xx}$ instead of y or one side of the Equation & $\sqrt{rr-yy}$ instead of $r-x$ on the other & we shall have $\frac{r\dot{x}}{\sqrt{2rx-xx}} = \frac{r\dot{y}}{\sqrt{rr-yy}}$.

Therefore $\frac{r\dot{x}}{\sqrt{2rx-xx}}$ is the fluxion of an Arc whose versed sine is x, of a Circle whose Radius is r, or more briefly thus, in the foregoing Figure $\mathrm{E}\!\left(\sqrt{2rx-xx}\right)$ is to $\mathrm{AC}(x)$ as $\mathrm{CB}(\dot{y})$ is to $\mathrm{Ec}\left(\frac{r\dot{x}}{\sqrt{2rx-xx}}\right)$ —

The Calculation of 1° Circumference of a Circle from the Line of 30° given ——— ——— ——— ———

If y be the Sin of any Arc, the Length of that Arc will be equal to $y + \frac{1*1}{2\times3}Ay^2 + \frac{3\,3}{4\times5}By^2 + \frac{5\times5}{6\times7}Cy^2$ &c. Consequently in a Circle whose Radius is 1, the Sine of $30° = \frac{1}{2}$ The Circumference of a Circle whose Diameter is 1, will be the following Infinite Series,

$$3 + \frac{1\times1}{2\times3}\times\frac{1}{4}A + \frac{3\times3}{4\times5}\times\frac{1}{4}B + \frac{5\times5}{6\times7}\times\frac{1}{4}C + \frac{7\times7}{8\times9}\times\frac{1}{4}D \text{ &c.}$$

whence we have ——— ——— ——— ———

A=	3, 0 0 0 0 0 0 0 0 0 0 0 0
B=	1 2 5 0 0 0 0 0 0 0 0 0
C=	1 4 0 6 2 5 0 0 0 0 0
D=	2 0 9 2 6 3 3 9 2 9
E=	3 5 6 0 3 8 4 1 1
F=	6 5 5 4 3 4 3 5
G=	1 2 7 0 9 5 4 4
H=	2 5 5 7 0 3 9
I=	5 2 8 7 9 9
K=	1 1 1 7 1 3
L=	2 4 0 0 5
M=	5 2 3 0
N=	1 1 5 3
O=	4 5 7
P=	5 8
Q=	1 3
R=	3
S=	1

3, 1 4 1 5 9 2 6 5 3 5 9 0

Of the Center of Gravity of Bodies

Tab. II. 5.
Fig. 3.
Fig. 85

Lemma

Let a, b, c, be any number of Bodies whose weights & places are all given, & let their centers be placed in y^e same right line, it is required to find their common center of gravity. Let the line a d (along the line in w^ch all the Bodies are supposed to be placed) be considered as an inflexible Mathematical Lever, supported at e but moveable about that Center. Then its plain that task is y^e Common Center of Gravity of y^e whole System. the whole System will rest upon that Point. Therefore the sum of the momenta, wherewith the Bodies a & b, endeavours to turn the Lever, one way, will be equal to the sum of y^e momenta wherewith, the Bodies, c & d, endeavour to turn it the other way. Let a x ea represent the momentum, wherewith the Body a endeavours to turn the Lever on the side of a, & the momenta of the rest will be b x eb, c x ec, d x ed. & we shall have the Equation a x ea + b x eb = c x ec + d x ed. Assume now some point as s, any where in the line a d, or rather in y^e Line a d produced so that the Bodies may all lye on the same side of s. Then we shall have ea = se - sa, eb = se - sb, ed = se - sd ec = se - se. Substitute those new values instead of the old ones in the former Equation, & it will be transformed into this, viz - a x se - a x sa + b x se - b x sb = c x se + d x sd - d x se.

Whl. Equation comprehended gives $x = \frac{ax + bx + cx + dx}{a+b+c+d}$ — Q.E.D. shows.

Multiply the weight of every body into its distance from the point, & divide the sum of the products, by the sum of the weights, & y^e Quotient will be the Distance of the common Centre of gravity from the point A.B.C.D.

Cor. 1. Hence if any number of Bodies, a. b. c. d. be ranged upon a Lever they will have y^e same effect upon it, as if all their weights were collected into their common Centre of Gravity.

Cor. 2. If we suppose the Lever to turn upon A, the sum of their momenta in y^e former Case, I mean placed at a Distance from each other, will be $ax + bx + cx + dx$ & their momenta when placed at e will be $a+b+c+d \times Ae$ and these 2 sums were found equals in the Demonstration of the foregoing Lemma.

Scholium. If the Point e be taken any where among the Bodies and not in y^e Line as produced, then must the sum of the products on the one side the common Centre of gravity & the sum of the products on the other side will be negative, add therefore according to the method for the adding affirmative & negative Quantities together, and the Centre of Gravity will be found to lie on the affirmative or negative

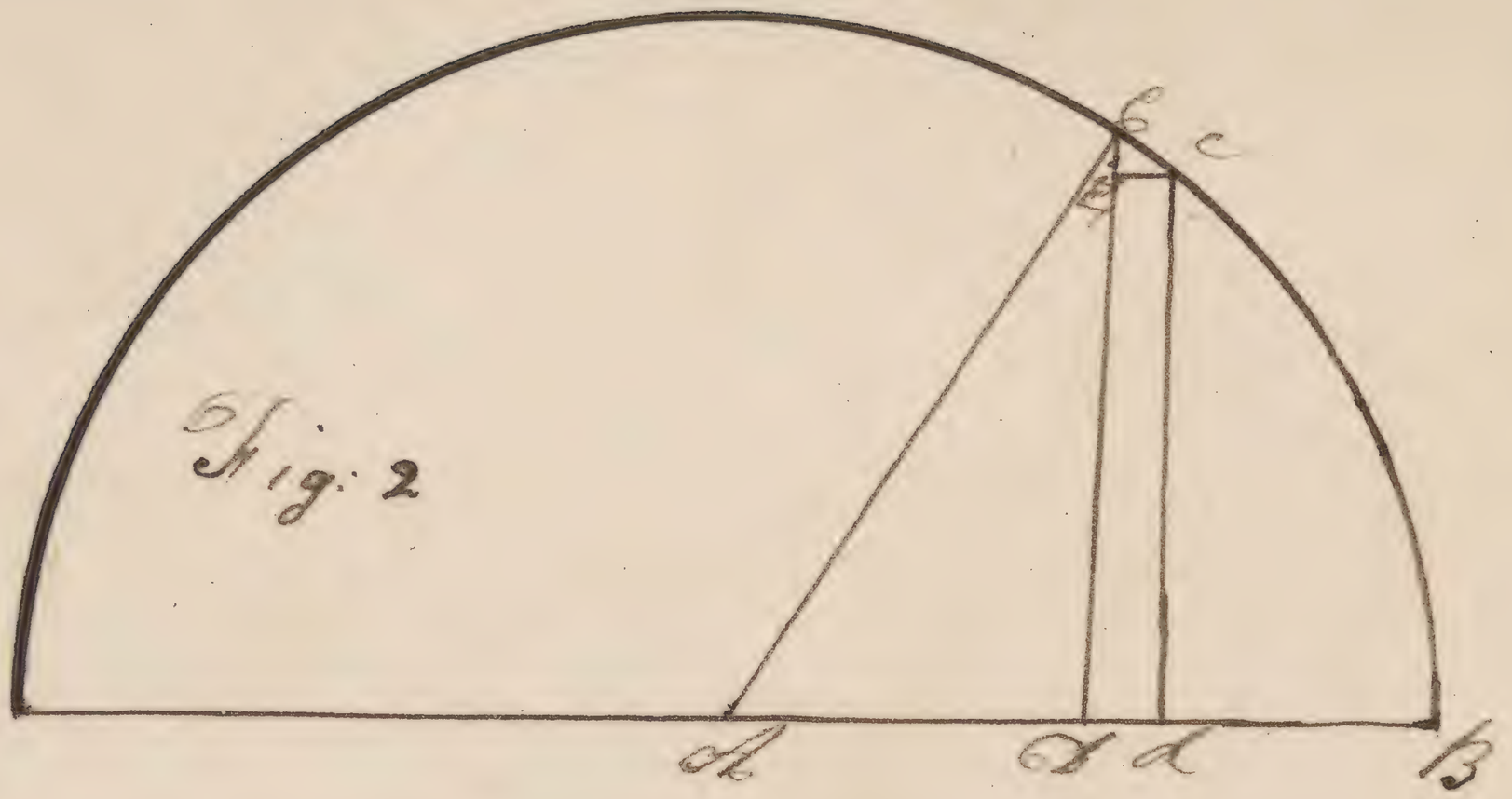


Fig: 2

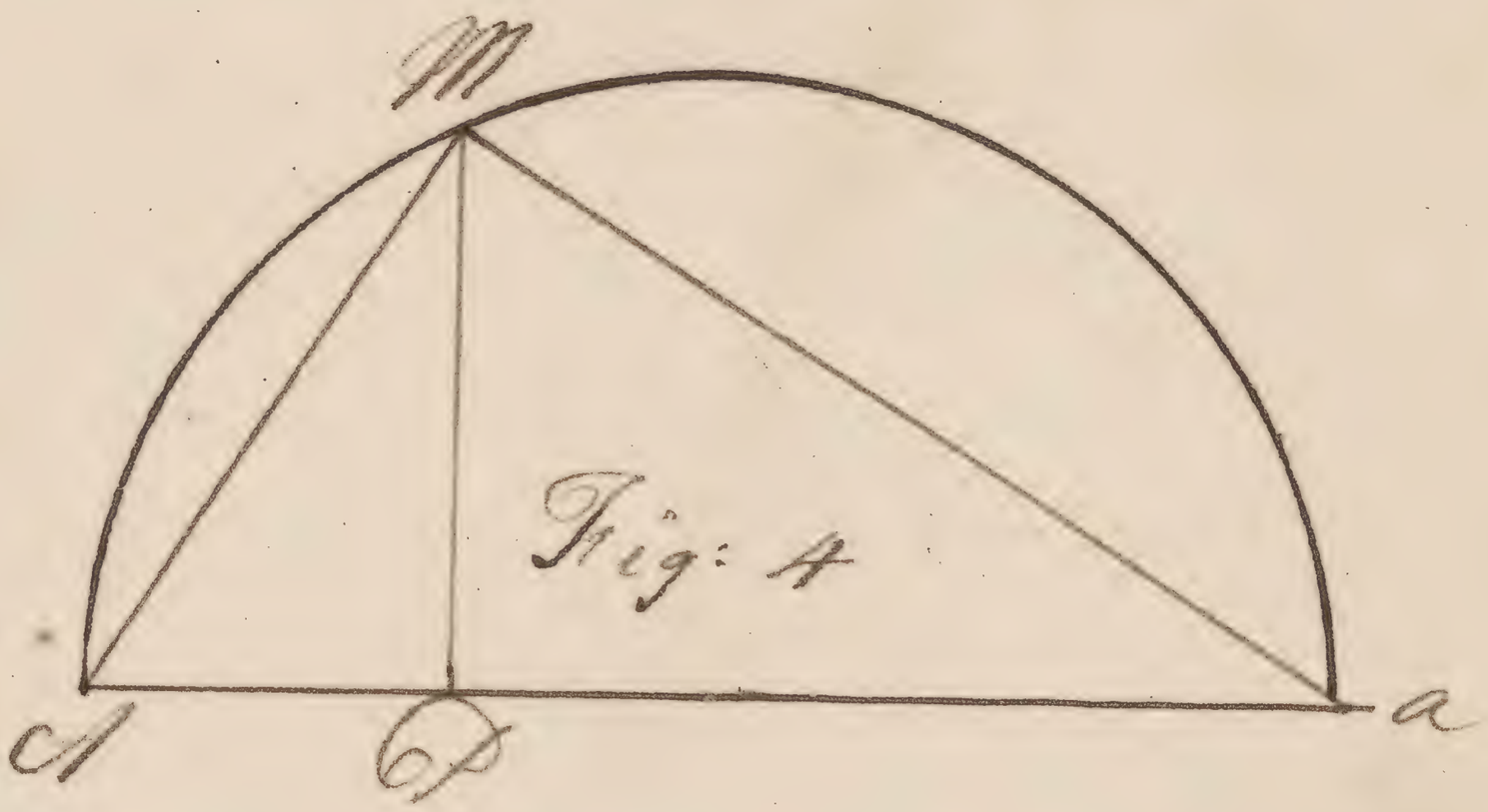


Fig: 4

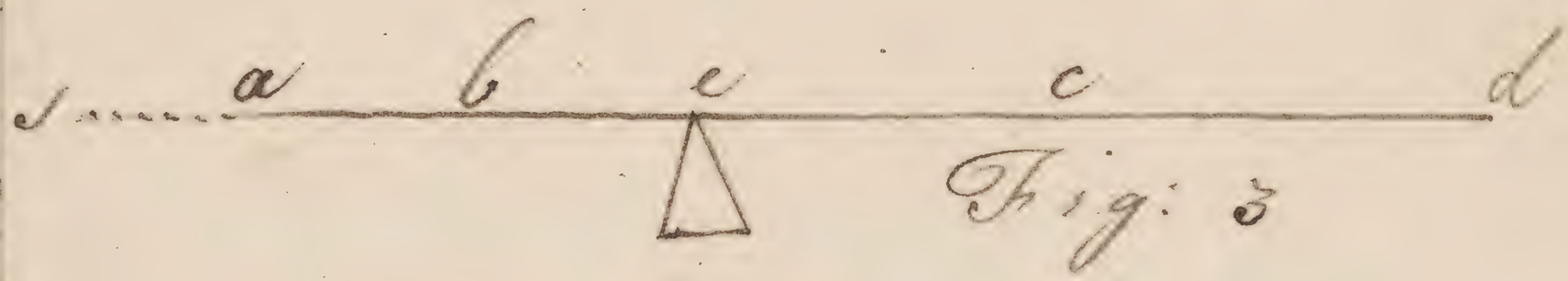


Fig: 3

Rod, according as the whole sum was affirmative or negative, if the whole sum be nothing the Point L will be the common Center of Gravity.

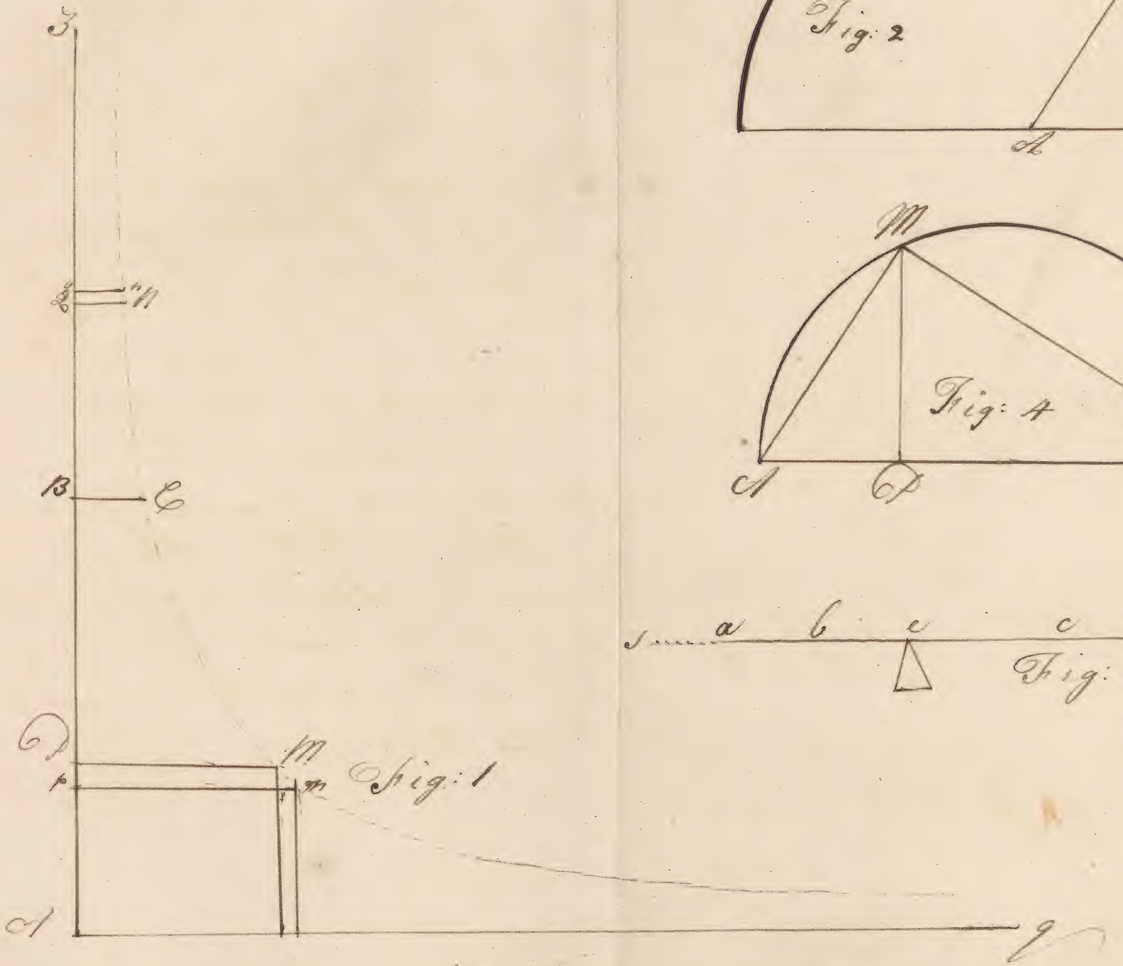
Instance. There is a Certain Cylindrical Rod, 36 Inches long, & weighing 1 Pound, at whose 2 Extremities are suspended 2 weights the one of 5 lb the other of 7 lb. Demand the common Center of Gravity of the Rod & those 2 weights. The Momentum of the Rod = the momentum of 1 lb at the Center. (by the Cor.) whence supposing 1 lb at the Place of the Pound & working as the Lemma directs, we shall have the Distance of the Center of Gravity from that weight = 15, 23 In.

Problem 16.

Tab. 6. Fig. 1 & 2 pag. 90.

To find the Center of gravity, of any Curvilinear Space ABC, that hath an Axis, & whose parts may be supposed endued with Gravity.

To find also the Centre of gravity of a Solid generated by the Revolution of such a Space about the Axis.



Imagine the whole Space ABC to be resolved into an infinite number of infinitely small narrow Trapezia, such as Mm, nn. whose respective centers of gravity are all in the Axis AD. & let all the Chords Mm, Nn, be at first supposed perpendicular to the Axis Call AP. PM. PN &c & supposing the weights of the several Trapezia, Mmnn. &c represented by their Areas to be multiplied into their respective Distances from the Perpendr. Call the sum of their products S. And call the whole Space ABC representing the sum of y^e weights of all its Parts. s. Then shall have $ds = 2yx\,dx$ & $s = \int 2yx\,dx$ Therefore the Fluents of those Quantities, i.e. of the Fluxions being found will determine the Quantities S & s or the Distance of the Cent. of Gravity from the Vertex. Let now the several Chords Mm, Nn. be supposed to turn upon their respective points P. so as to become inclined to y^e Axis, but still parallel to one another. & the Centre of gravity of the whole Space ABC will still continue in the same place, for tho' by this means y^e weights of all the Parts be diminished, they are diminished in the same proportion. Lastly let all the Chords Mm, Nn be supposed to return again into their perpendicular positions. & let the Space ABC generate a Solid about its Axis AD & we suppose the Diameter of a Circle to be to y^e Circumference as 1 to π & call the Quantities S & s as before we shall have in this Case $\dot{S} = \pi yyx\dot{x}$ & $\dot{s} = \pi yy\dot{x}$ Whence S & s may be determined as before. — — — —

Ex. 1. Let the Nature of the Curve be expressed by this Equation, $px x^{\frac{m}{n}} = y^n$ then we shall have $y = p^{\frac{1}{n}} \times x^{\frac{m}{n}}$ & $2y\dot{x}$ or $\dot{s} = 2p^{\frac{1}{n}} \times x^{\frac{m+n}{n}} \times \dot{x}$ Whence $s = \frac{2n}{m+2n} \times p^{\frac{1}{n}} \times x^{\frac{m+2n}{n}}$ or $\frac{2nx}{m+2n} \times xy$. moreover $\dot{s}$ or $2y\dot{x} = 2p^{\frac{1}{n}} \times x^{\frac{m}{n}} \times \dot{x}$ Therefore $s = \frac{2n}{m+n} \times xy$. Therefore $\frac{t}{s} = x \times \frac{m+n}{m+2n}$

Ex 2. For the Centre of Gravity of y^n &c we shall have, according to the foregoing Notation $2yy\dot{x}\dot{x}$ or $\dot{s} = 2p^{\frac{2}{n}} \times x^{\frac{2m+4n}{n}} \times \dot{x}$ Therefore $s = \frac{n}{2m+2n} \times 2p^{\frac{2}{n}} \times x^{\frac{2m+2n}{n}} = \frac{nx}{2m+2n} \times 2yy\dot{x}$. Again $\dot{s}$ or $2yy\dot{x} = 2p^{\frac{2}{n}} \times x^{\frac{2m}{n}} \times \dot{x}$. Ergo $s = \frac{2n}{2m+n} \times 2yy\dot{x}$. Therefore $\frac{t}{s} = \frac{2m+n}{2m+2n} \times x$ And the Distance of the Centre of gravity of y^2 whole Solid will be the Axis x multiplyed into the fraction $\frac{2m+n}{2m+2n}$.

Scholium —

As the two former Examples, will give the Solution for this Problem, in an infinite number of Curvilinear figures, so it will be Triangles & Parallelograms, & the Solids, generated by their Revolutions about their Axis, viz. Cones & Cylinders. for in a Triangle $bx = y$ & in a Parallelogram $b = y$ & $px^0 = y$!

Tab: 6
Fig: 2
Page 90

Theorem -

If in a Triangle as ABC from any Angle as A, be drawn a line as AE bisecting y^e opposite side BC, & if upon that line be set off AF = $\frac{2}{3}$ of AE the Point F will be the Centre of gravity of y^e Triangle

For as the Line AE, bisects the Line BC it must bisect every other Line parallel to it, & cons. the Centre of Gravity must be some where in y^e Line AE. for y^e same Reason, if the Line BD be Drawn bisecting AC the Centre of gravity must also be somewhere in BD. therefore it must be in y^e common intersection F join DE which as it bisects both AC & BC will be parallel to AB. & we shall have from similar Triangles, AF. FE :: AB. DE :: BC. BE :: 2. 1 & Componendo AF. AE :: 2. 3.

Tab: 6.
Fig: 3.
Page 90

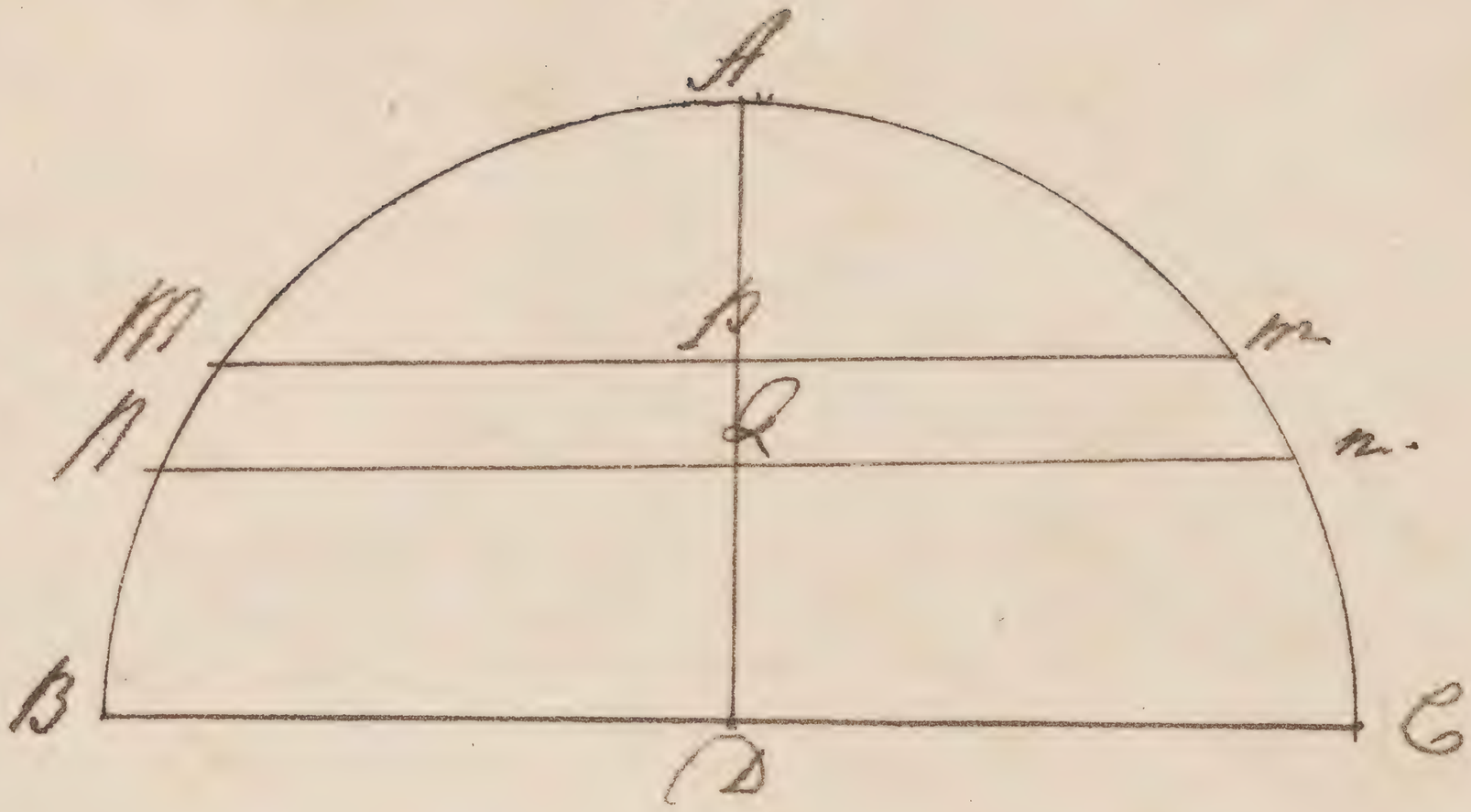
To find the Centre of gravity of an Hemisphere generated by the Revolution of a Semi-Circle ACB about its Axis AB. Here it will be proper for a more simple Calculation to Call AP. x & MP. y as before & if we call the Radius r, we shall have $yy = rr - xx$ & $2yy\dot{x}$ or $s = 2rrx\dot{x} - 2x^3\dot{x}$ Therefore in the Case of the Solid generated by the Area MBC m. we have $\frac{1}{2}$ $\overline{2rrx - 2x^4}$ & in Case of y^e whole Hemisphere $s = \frac{2r^4}{2} - \frac{2r^4}{4} = \frac{2r^4}{4}$

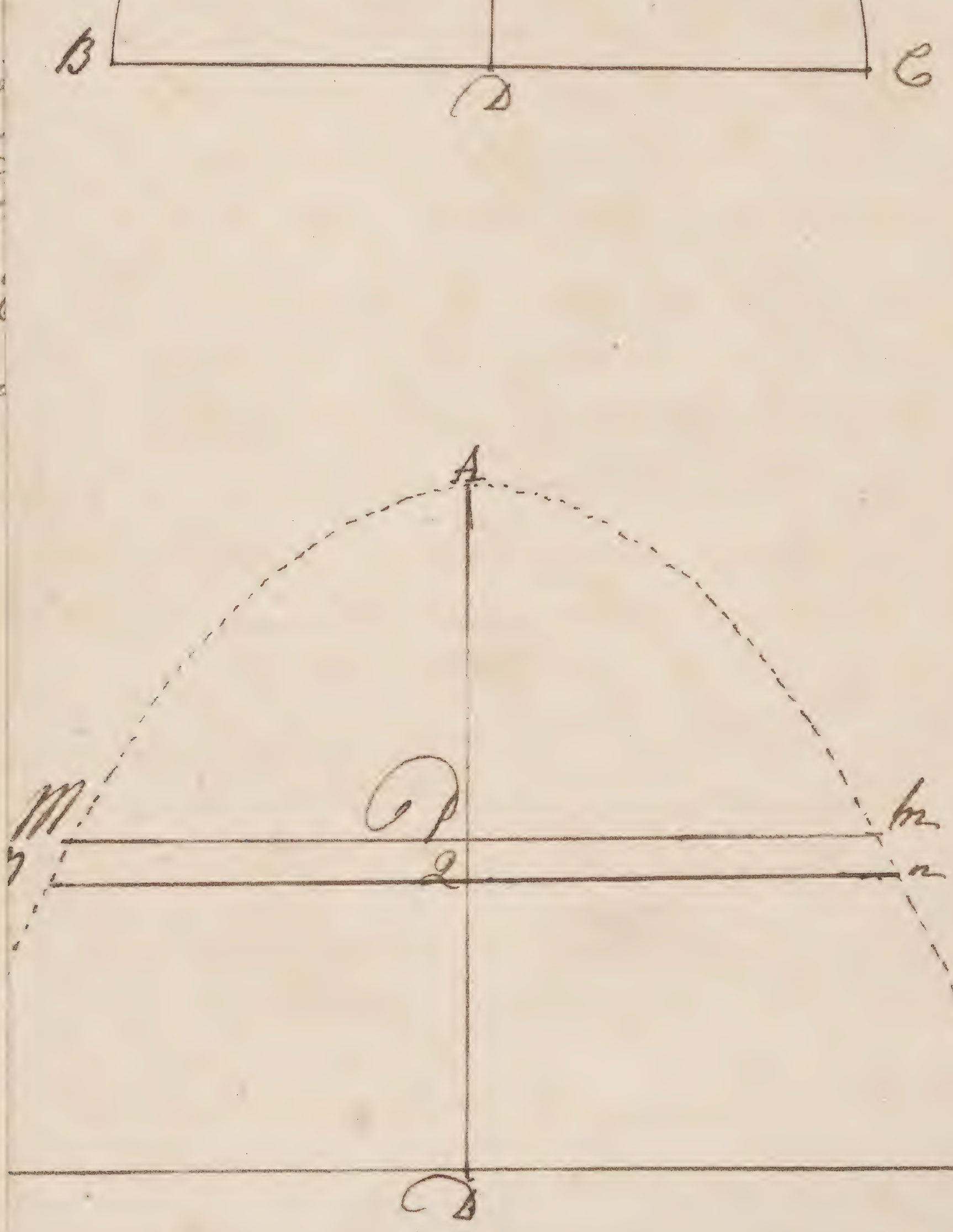
Again s or $2yy\dot{x} = 2rr\dot{x} - 2xx\dot{x}$ & yc $s = 2rrx - \frac{1}{3}x^3$ which in the Case of y^e whole Hemisphere $= 2r^3 - \frac{1}{3}2r^3 = \frac{2}{3}2r^3$ $s = \frac{3}{8}$ that is in an Hemisphere or Hemisphered the Centre of gravity is $\frac{3}{8}$ of the whole Axis, from the Base, for if all the Elementary Lamina be conceived in a given Direction, they will form an Hemisphered Solid —

That if all the Elementary Lamina of a Sphere be contained in a given Proportion they will form an Spheroid, may be proved from Prop. 32 Lib. 12 or thus. Let an Ellipse ABab (Fig. 4. Page 364 Tab 8) & a Circle DEde have the same Common Axis Aa & Let DMm be an Ordinate to the Axis, meeting the Ellipse in M. & the Circle in N. I say then that y^e DM. will be to PM. always in y^e same Proportion, (viz.) that of CB to CD for from the Nature of the Ellipse DM^q. : DM × Da :: CB^q. : CA^q. :: CB^q. : CD^q. & y^e DM^q. of DM × Da :: CB^q. : CD^q. But DM × Da = DM^q. from y^e Nature of y^e Circle wherefore DM^q. : PM^q. :: CB^q. : CD^q. :: CB^q. : CD^q. Therefore if we imagine the Circle and Ellipse to turn round their common Axis, so y^t the Circle may generate a Sphere & the Ellipse a Spheroid, & then these two Solids be resolved into an infinite number of thin Lamina, every Lamina in the Sphere will be to its correspondent Lamina in the Spheroid as BC^q. : CD^q. for if DM & PM be the semidiameters of 2 correspondent Lamina, these Lamina will be to one another, as DM^q. : PM^q. i.e. as the Squares of their semidiameters But DM^q. : PM^q. :: CB^q. : CD^q. Therefore if all the Elementary Lamina be contained (in this Semisphere) in a given Proportion they will form an Semisphere.

Ex 4.

Tab: 6: Page: 90

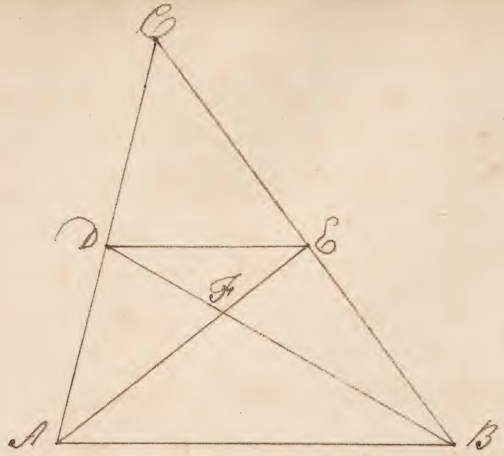


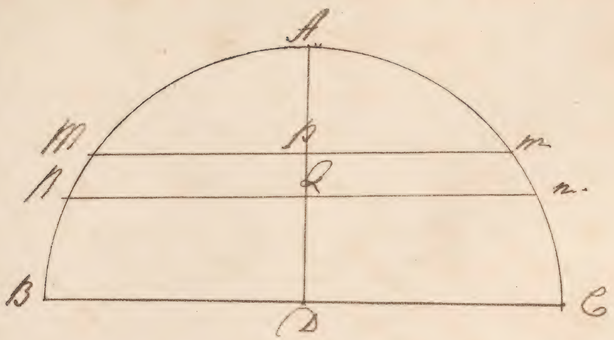


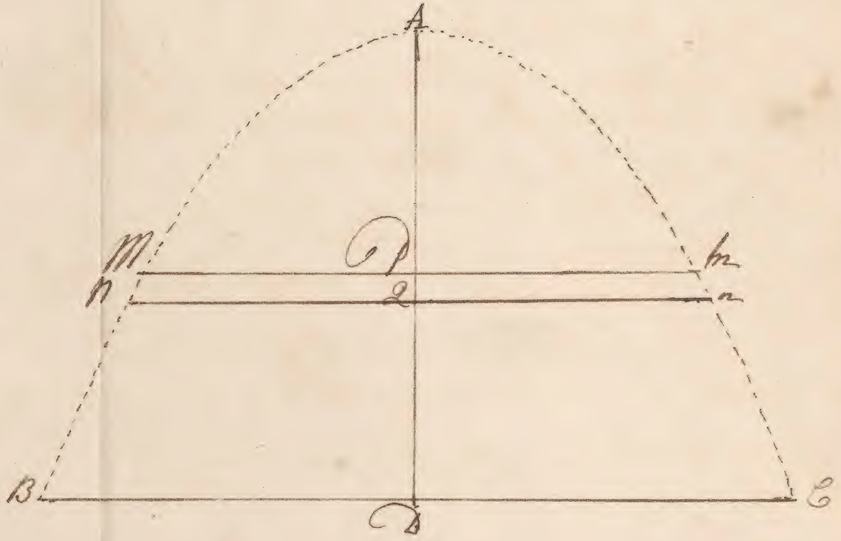
Ex 4.

Let us in the next place enquire in y^e Center of gravity of the generating Semi Circle, & here since $yy = ax - xx$ we shall have $-2xx\dot{x} = 2y\dot{y}$ & $2yx\dot{x}$ or $s = -2y\dot{y}$ whose fluent $-\frac{2y^3}{3}$ wh: $x = *$ will be $-\frac{2a^3}{3}$ & whence $x = r$ will be $* + \frac{2r^3}{3}$ because whilst x flows from a to be = the fluent flows from $-\frac{2a^3}{3}$ to be = a therefore in this time the fluent has got $\frac{2r^3}{3} *$

Therefore $s = \frac{2r^3}{3}$ but s is the Area of a Semicircle $ABC = \frac{arr}{2}$ therefore $\frac{s}{3} = \frac{4r}{3a}$ which is as much as to say, that as 3 times the Circumference of any Circle is to 4 times the Diameter, or as 33 to 14 very nearly, so is y^e whole Axis of any Semicircle or Semi Ellipsis to y^e Part between the Base & Centre of Gravity. — — —







9

Prob. 4

To find the weight of an infinite Tall Prism or Cylinder, standing upon the surface of the Earth, upon a supposition, y^t the force of gravity, in all Places is reciprocally as y^e Squares of y^e Distance from the Center of y^e Earth.

Let the Cylinder in Question be distinguish'd into an infinite number of thin Lamina's, & let x be the thickness of one of these thin Lamina's at any indeterminate Altitude x from the Center of y^e Earth. Then will the weight of this as $\frac{x}{1}$ i.e. $\frac{x}{r}$ will be the Fluxion of the weight of y^e Cylinder whose Fluent is $\frac{x}{2r}$. Call the semidiameter of y^e Earth r, and the Value of this fluent at y^e Earths surface will be $\frac{1}{2}$ & at an infinite Distance, $\frac{1}{\infty}$ or haply will be nothing. Therefore the weight of this infinite Cylinder will be $\frac{1}{2r}$ & $\frac{1}{2}p$ will be the weight of a small part of this Cylinder, at the surface of the Earth, whose Altitude is a, and this weight according to the former proportion, ought to be represented by $\frac{a}{rr}$ therefore, the weight p will be to the weight of the infinite Tall Cylinder as $\frac{a}{rr}$ to $\frac{1}{r}$ as a to r.

* $\frac{1}{2}vv$ will be $\frac{1}{AB} - \frac{1}{AA}$. for when $x = AA$ (1 & when v & conf. $\frac{1}{2}vv = *$) the fluent $\frac{-1}{x} = \frac{-1}{AA}$ & when $x = AB$, the fluent $\frac{-1}{x} = \frac{-1}{AB}$ therefore while one fluent has got $\frac{1}{2}vv$ the other fluent (& they flow equally) has got from $\frac{-1}{AA}$ to become $\frac{-1}{AB}$ i.e. it has got $\frac{1}{AB} - \frac{1}{AA}$.

Prob. 12

Tab. 7
Fig. 3.
Page 110

Let S be the Centre of the Earth, B any Point in its Surface, & let the force of gravity in all places be reciphocally as the Squares of their Distances from the Center of the Earth. It is required to determine the Velocity of a heavy Body, at y^e Surface of y^e Earth, which it acquires by falling from a given Altitude at B——I

Let x be an indeterminate Distance from the Center of the Earth & let v be y^e Velocity of y^e falling Body, at y^t Distance, Let $\frac{1}{xx}$ represent the Force of Gravity at B & Conf. $\frac{1}{xx}$ its force at the Distance x. & thon its plain y^t after y^e falling Body is arrived at y^e Distance x & thon descends further, thro' an infinitely small Space $\dot{x}$, y^e time of y^e infinitely small Descent, will be as $\frac{\dot{x}}{v}$ & it will be as y^e Space directly & as the Vel. inversly, in y^e Descent, will be as y^e time $\frac{\dot{x}}{v}$; & the accelerating force $\frac{1}{xx}$ by y^e Def. that is $\dot{v}$ will be as $\frac{\dot{x}}{xx}$ therefore the fluxion of the Velocities arising from these two fluxions that are generated in equal times will be proportionable that is $\frac{1}{2}vv$ will be as $\frac{1}{SB} - \frac{1}{SA}$. or as $\frac{SA - SB}{SB \times SA}$ or as $\frac{AB}{SB \times SA}$.

Therefore because by the Constant Quantities 2 & SB, vv will be as $\frac{AB}{SA}$. This being drawn into SB by the Height from whence a Body will fall to the Surface to y^e Earth in y^e first of Time, & since during so small a descent, the force of gravity, may be look'd upon as uniform 'tis evident that a Body falling from A to B.

will acquire a Velocity v. will carry it uniformly thro the space 2 BD in 1" of time.
Let 2 BD represent this Vel: then must any other Vel: be represented by thro spaces which it will carry a Body in 1" of Time. Now to find the Vel: acquired in falling from A to B.
I say as $\frac{DB}{AB} : \frac{AB}{AB} :: \frac{1}{2}DB : \frac{1}{2}$ the Sq: of the Vel: acquired in falling from A to B. $:: \frac{1}{2}DB \times \frac{1}{2}DB \times AB$ the Sq: of the Vel: acquired in falling from A to B $= \frac{1}{4}m^2 \times \frac{AB}{AB}$ supposing m a mean proportional between DB & 1. Therefore a Body falling from A to B. acquireth a Vel: v. would carry it uniformly thro' the space $2m \times \frac{AB}{AB}$ in a second of Time & &.

Scholium.

DB is found by Experiment to be $15\frac{1}{2}$ Paris feet. & 1 196 158 $15\frac{1}{2}$ of 4° same measure therefore 2m is about 4 English Miles.

Cor. 1. If AB be infinite the Quantity $\frac{AB}{AB}$ = 1 & therefore goes out of the Question, therefore a Body falling from an infinite Height will acquire at the surface of ♁ Equ: but a finite Velocity, (viz) such a Vel: as will carry a Body moving uniformly thro' 8 Miles in 1" of Time.

Cor. 2. Therefore a Cannon ball of a Body be thrown up with such a Velocity, it will never return but ascend ad infinitum.

Cor. 3. After the same manner, may be 95
determined, y^e Col: of a falling Body, w^ch
be the Law of Gravity ——

Lemma

If two Bodies fall from equal Altitudes their Velocities at all their equal Altitudes will be equal, tho' one descends obliquely, & the other perpendicular.

Demonstration.

Tab 7 Fig 1 Pag: 110

Let ABC be an Horizontal Chord of any Curve ADC whose lowest Point is D, & whose Axis BD is perpendicular to the Horizon, Let E be any Point in the Curve AD, and e another infinitely near it, & draw the Horizontal Chords EF & ef cutting the Axis in F & f respectively; then imagine a Body as P to descend [illegible] at A along the Curve AD, and another as Q to fall perpendicular from the Point E in the Axis BD, & let the Space E be so taken, that the Vel: of the Body Q in y^e Point E acquired in falling from E to F may be precisely equal to the Vel: of the Body P in y^e Point E acquired in falling from A to E; & let e be so taken, that the Vel: of the Body Q in y^e Point e acquired in falling from E to e may be precisely equal to the Vel: of the Body P in y^e Point e acquired in falling from A to e obliquely: Now in the time that infinitely short Space Ee is described with the Vel: the Space Ff is described in the time Ef, & if the Space is considered as an Horizontal Plane, it is further evident from the nature of such a Plane, that the Force wherewith the motion of the Body P in E is accelerated in falling from E to e, is to that Force wherewith the Body Q is accelerated in falling from E to F respectively; as Ff to Ee Therefore the time Ee is described in, is a much greater then the time Ff is described in, as

the mis accelerations f. is greater than the mis accelns: in ℓ. Therefore the Increments
and to the velocities of the Bodies f & g
in falling from a to ℓ & from f to h, shall
be as their respective times & forces conjoined
together, will be equal. Therefore the Velocities
of the Bodies f & g in the Points ℓ & h will
still be equal. i.e. if the Vel. of the Bodies
f & g in falling from a & A be in any one
particular Case equal, at equal Altitudes, they
will always be so, that is the Incrt of the Velocity
of f = to the Incrt at the Point ℓ & the Velocity
at ℓ = to the Incrt at the Point h & the Velocity
at the Point h. Therefore if 2 Bodies fall from
equal Altitudes, their Velocities at all the other
equal Altitudes; will be equal. tho' one descends
all the way by the other. Dependiendo

Cor. 1. The Vel: of the Body is in the Point 99
C, will be in a subdupl: Ratio of BF the Distance
between the Horizontal Lines of BC & DF; because
the Vel: of the Body G in the Point F is to
Cor. 2. If the Arc AC be part of a Circle whose
Diameter is DG, the Vel: of a Body in the lowest point
D, again in falling thro' the Arc AD, will be to
Vel: of a Body in the same Point falling thro' the
Arc ED, as the Chord of the Arc AD to the Chord of
the Arc ED, for the former Vel: is to the latter
as $\sqrt{BD}$ to $\sqrt{BF}$ or as $\sqrt{DG \times DB}$ is to $\sqrt{DG \times DF}$ but
$\sqrt{DG \times DB}$ is the Chord of the Arc AD, & $\sqrt{DG \times DF}$ is the
Chord of the Arc ED.

Prob: 13.

To find practically the time of a least Oscillation
of a given Pendulum, swinging in an Arc of a Circle
& to find it without any Sensible errour, also the time
of any other —

Retaining the same Construction of y^e last
Lemma & supposing, moreover, that the Curve AC is
an Arc of a Circle whose Diameter is DG, wherein
the Pendulum performs its several Oscillations, being
now supposed to be in the Point C, in its Descent
from C to D Vel: $\sqrt{BF}$ expresses the Vel: acquired
by any heavy Body in falling from B to F, &
Cons: the Vel: of the Pendulum at y^e Point C,
shall will $\frac{1}{2}\sqrt{BD}$ expresse the Vel: acquired in falling
thro' a Space equal to $\frac{1}{4}$ BD. Wherefore a Body by
this Vel: would be carried uniformly thro' a Space
equal to $\frac{1}{2}$ BD in the same time time it would fall
thro' $\frac{1}{4}$ BD, divide the Space $\frac{1}{2}$ BD by the Vel:
$\frac{1}{2}\sqrt{BD}$, & let the Quotient $\sqrt{BD}$ expres the time
wherein a heavy Body would fall thro' $\frac{1}{4}$ BD, i.e. B
thro' half the Length of the Pendulum;

* See Lemma at y^e End of this Chap.

Then will $\frac{Le}{\sqrt{B.F.}}$ express the time wherein the Arc Le is described by the Pendulum. But then the Arc Le is the flexion or increment of the Arc DLL by which the q. Diab. $= \frac{1}{2} \frac{Dd \times Fh}{\sqrt{Fh \times hD.}} = \frac{1}{2} Dd \times \frac{Fh}{\sqrt{Fh \times hD}}$. But if B Din H K h D in L And if the Arc d D C be not very large the Lin* L. cannot differ sensibly from its middle Ch, nor $\frac{\sqrt{D}}{\sqrt{dh}}$ from $\frac{dL}{dH}$ Therefore $\frac{Le}{\sqrt{Bh}}$ is equal very nearly to $\frac{dL}{dH} \times \sqrt{D} \times \frac{\frac{1}{2} Fh}{\sqrt{Bh \times hD}}$. Upon the Diameter Bh described a Circle BGhDB cutting the ChD, & Le in G & g, respectively, & $\frac{\frac{1}{2} BD \times Fh}{\sqrt{Gh.Bh \times Fh.D}}$ will be equal to Gg. The Flexion of the Arc GhD. Therefore $\frac{\frac{1}{2} Fh}{\sqrt{Gh.Bh \times Fh.D}} = \frac{Gg}{BD}$. Therefore $\frac{Le}{\sqrt{BFh}}$ or the time wherein the Pendulum ascends from L to e will be $\frac{dL}{dH} \times \sqrt{D} \times \frac{BD.Gg.GB}{BD}$. Therefore the whole time of its descent from B to L will be $\frac{dL}{dH} \times \sqrt{D} \times \frac{BD.Gg.GB}{BD}$ & the time of an Entire Oscillation thi' the Arc dL will be $\frac{dL}{dH} \times \sqrt{D} \times \frac{BD.Gg.GB}{BD}$. Let the Arc dL be infinitely small & the Quantity $\frac{dL}{dH}$ will become equal to Unity & the time of an Oscillation will be $\sqrt{D} \times \frac{BD.Gg.GB}{BD}$. That is the Diameter of any Circle is to the Circumference so is $\sqrt{D}$ the time of an heavy Body's fall from rest the half the Length of the Pendulum to the time of an Oscillation of the Pendulum. Call the time thus found t. & if the Arc dL be not very large the time of an

The time of a beat of Oscillation is to the time of an Oscillation in a small Arc, as t. to $t \times \frac{KD}{KL}$ or as KD to KL or as HG to KL Let the Arc HDE be a Semicircle whose Diameter is 1. then will $KD = \frac{1}{2}$ $HK = \frac{1}{4}$ $LD = \frac{1}{8}$ $HL = \frac{3}{4}$ or $\frac{6}{8}$ $KL = \frac{7}{8}$ Therefore $HK : KL :: \frac{6}{8} : \frac{7}{8}$ or as $6 : 7$.

of Gravitation the' that Area will be $t \times \frac{gk}{gk}$ or 1 /
it will be $t \times t \times \frac{gk}{gk}$ = 20 ft

Scholium

If the Gravitation were a constant Power in the least Gravitation, and be to the time of Vibration the' that Isochronic according to the Position above laid down as 6 to 5 or as 29 to $33\frac{5}{6}$ ft by other means we know, that the Consequent of this last Position ought to be $34\frac{2}{3}$ ft $\frac{1}{30}$ ft greater than $34\frac{1}{30}$ If therefore the Experiences above mention approaches so near the truth in so large a Pendulum as a Foot Pendulum it is a plain Argument that in an Area infinitely small, it will be easy to conjecture how exactly Parallel the Excursions will be in small Arcs such as are usually described by Pendulums.

Cor. 1. If two Pendulous Bodies of the same figure & magnitude be together with their Centers of Gravity & of equal Distances from the point of Suspension suffer their Oscillations, in the same time, in an Argument that the Quantity of Matter of both the one & Pendulums are the same: & conf. that the Quantity of matter in bodies gravitating unto the Center of the Earth, will be always proportionable to their weights for if the one Pendulums, were to be less than the other, they would fall slower, & in longer times, & Conf. would be equally accelerated.

Cor. 2. The Space through which a heavy Body falls from rest in a given time, is to half the length of the Pendulum that oscillates in that time as the Square of the Circumference of any Circle

is to the square of the Diameter. For as 4° two first Terms in the Proportion are Spaces, the two last represent the Squares of the Times in which those Spaces are described.

Cor. 2. Small Vibrations of 4° same Pendulum are made, per found in the same time nearly for 4° small Quantity of time $\frac{1}{4}K$ which is to be added to it the time of a least Oscillation wch in this Case makes no sensible variation in the whole.

Cor. 3. The Length of any Pendulum is as the Space of 4° the time of a least Oscillation, & of Pendulum multiplied into the absolute force of gravity; for any heavy Body falls thro' any Space as t^2 in any time, the Space I will be as the Square of that time × the force of gravity. But the time of Descent and this again as the time of a least Oscillation of a Pendulum, whose length is 6 for 4° same reason as the Circumference of a Circle is as the Diameter. Therefore the length of any Pendulum, is as the Square of the Times of least Oscillation of that Pendulum, × the absolute force of gravity

NB. If this Corollary be applied to the least Oscillation of a Pendulum upon an Inclined Plane instead of the absolute force of gravity, understand that force, wherewith the Body, endeavours to descend, in a right Line along the Plane

Cor. 5. Therefore the time of a least Oscillation of a Pendulum, is as a subdup. Ratio of it's Length & Pendulum directly, & in a subdup. Ratio of it's force of gravity inversely. for if we encrease the Length of any Pendulum, & take the force of gravity, of a least Oscillation, it's will be as before by the last Cor. 4: therefore it will be as $\frac{\sqrt{l}}{v}$

Cor 6. Therefore if the time of a least Oscillation be given viz. $\frac{\sqrt{l}}{v}$ & long. $\frac{l}{v}$ will be a given Ab. in w. it will be as l. i.e. the force of gravity will be every where as the Length of an Isochronous Pendulum.

Cor. 7. The force of gravity being given the time of a least Oscillation will be in a Subdup. Ratio of it's Length of that Pendulum.

Cor. 8. Universally if v be the force of gravity & the Length of any Pendulum, & n any number of vibrations of that Pendulum, & t it's time wherein the number of vibrations is performed the Quantity v/t will always be as the Ab. Ln. so if $\frac{\sqrt{l}}{v}$ represent it's time of a single Oscillation then will $\frac{\sqrt{l}}{v} \times n$ represent the time of any number of these Oscillations. That is, the time t will be as $\frac{\sqrt{l}}{v} \times n$, therefore t will be as $\frac{Ln.}{v}$. & v as Ln.

Cor. 9. The times of similar Oscillations (by w.h
is meant such as are made in similar Arcs) will
always be in a subdup. Ratio of the Lengths
of the Pendulums, w.h were the D.E. of those Arcs,
provided the force of gravity continues y.e same.
for if the Arcs A D & a d. & D & Comp. be like
as the Diameter D d. the Line B F & b f will be
so too, & $\frac{De}{\sqrt{DF}}$ a.y.e Temperature in the Arc is described
will be as $\frac{De}{\sqrt{DF}}$ or as $\sqrt{DF}$ & comprehend y.e whole
time of an Oscillation this: the whole Arc d e
will be as $\sqrt{Df}$ or as $\sqrt{\frac{Df}{2}}$ i.e in a subdup.
Ratio of the Lengths of y.e Pendulum. ———

Cor. 10. The force whereby any Pendulum whose
length is l is moderated, in any Point & Situ
Arc. will be as $\frac{DF}{l}$ for if the weight of y.e
Pendulum & the absolute force of gravity, be
called 1, the Accelerating force will be at E,
$\frac{DF}{De}$ but $De = \frac{1}{2} \frac{2l \times DF}{\sqrt{2DF \times DF}} = l \times \frac{DF}{2DF}$ ———
Therefore $\frac{De}{DF} = \frac{l}{2DF}$ & $\frac{DF}{2e} = \frac{2DF}{l}$ ———

Cor 11 Therefore is made of the Motion & force wherewith the same pendulum is accelerated in its motion, will always be as its distance from the lowest Point of its Arc. so in this case the line EF defines nothing for the Arc DE

Cor. 12. If the Arc DE be supposed infinitely small, & velocity of the same pendulum, at any points E, will be as $\sqrt{DA^2 - DE^2}$ for this vel: will be always, as $\sqrt{BF}$. i. e. as $\sqrt{BD - FD}$ that is in the same Circle as $\sqrt{BD \times DI - FD \times DS}$. But $BD \times DI$ is $\frac{1}{4}$ Square of the Chord of $\frac{1}{4}$ Arc AD, & so $\frac{1}{4}$ same Reason $FD \times DS$ is the $\frac{1}{4}$ of the Chord of $\frac{1}{4}$ Arc ED. Therefore the Vel: of the Pendulum in the Point E is as $\sqrt{DA^2 - DE^2}$

NB. The following Theorem with its Corollaries are added for y^e better understanding of what is delivered in the Remarks, concerning the motion of fluid Bodies and y^e Curves described by y^e Centripetal

Theorem

Tab: 1 Fig: 2 Pag: 110

If any Body as B be made to gallate in a right line, A D, whose middle point is D. Being accelerated in its motion from A to D. Equally retarded in its motion from D to C. By forces any where proportionable to the Distance of the Body from the middle point D. I say then that the Body B will perform its Gallations in y^e same time whatever be the length of the line A D C.

To Demonstrate this Imagine, two such Bodies P & Q. beginning their Gallations at y^e same time & both Gallating in the same line P in a larger Space A D C. & Q in a smaller, a Dc. Then 'tis plain y^t the Accelerations and Velocities of those two Bodies in correspondent Spaces shall be in any given time will always be in y^e same proportion. to wit in the proper. of D A to D a. Therefore the Bodies P & Q will take same to the point D at the same times & Consequently both perform their Gallations in y^e same time. For the latter concerning the Forces, whereby the 2 Bodies P & Q are agitated to act not continually, but by successive finite impulses at equal intervals of time, and always proportionable to y^e Distances of the Bodies from the Centre D. And then Imagine those impulses &

Intervals be the Spaces D ult. misimilum.
Cor. 1. Satisfying the Body D to Gallate in
Lineo it be as a bone, or to be any where
accelerated or retarded, by a force ult. is Leg° absolute
force of gravity as the Distance D ito to equi-
Lineæ or as $\frac{DP}{d}$ be 1. I say then, that the
time of an Oscillation. Ito; the Space A D will
be equal to the time of a least Oscillation of a
Pendulum, whose Length is L. For supposing
the Line A D C to be infinitely small, and
supposing a Pendulum whose Length is L to
swing in an Arc = to the A, C, or Line
A D C, the vis acceleratrix of the Body
P, at any Distance D P, will be equal
to Leg° Ito its acceleratrix of the Pendulum, et eo
same Distance from the lowest point of its Arc
(by the 10 & 11 Coroll. of Leg° last Prop.) Wherefore
their Accelerations and velocities will correspondent
Points will be equal, & Conf. also the times
their Oscillations. But the time of an Oscilla-
tion, Ito the Line A D C, will always be
the same with the time of an Oscillation
Leg°. universally, the time of an Oscillation
Ito the Space A D will be equal to the time
of a least Oscillation of a Pendulum whose Length
is L ———————

Cor. 2. Since the force wherby the Body is acce-
=lerated, or retarded, is any where as $\frac{DE}{L}$ (Conf.
as $\frac{L}{L}$; if the Distance D Degrees in L^q it follows,
that if the accelerating or retarding forces, in every
Point of the Line AE be proportionably increa-
or remitted, the Length L will be reciprocally as the
force at any given Distance. But the Time of Oscil-
-lation of Pagrum's pendulum, whose Length is L
is conf. the Time of an Oscillation thro' the Space
ADE is as $\sqrt{L}$ by the Cor. 1 Coroll. last pro.
Therefore the time of an Oscillation thro' the
Space ADE is reciprocally as a subduplicate Ratio
of the accelerating or retarding force at any
given Distance from the Point D.

Cor. 3. By the Demonstration of ye foregoing
Theorem it appears, that if a Body as B Oscillates in
a larger Space ADC and another as b. in a
smaller contracted Space a De, & if the Distances
DB, De be taken in ye same proportion, and it be
that then the Velocity in D will be to Vel: in
e. DA to De. Therefore if the Space ADC
be sometimes enlarg'd & sometimes contracted,
(the accelerating forces at ye same Distances continuing
always the same) And if the Distance DB be
always taken in ye same proportion to DA: the
Vel: at the Point D will always be as DA.

Cor 4. Those things being supposed as also y^t the accelerating & retarding forces, at the same Distances from D. continue always the same; If upon the line AE as a Diameter be described a Semicircle AME, & a Line as MP be drawn any where perpendicular to AE in P. I say that the Velocity of the oscillating Body in the Point P will always be as y^e portion Diameter AE. For if we suppose the Line AE to be diminished in any proportion, that the Line DA is to decrease in the same proportion, as $\sqrt{DA^2 - DP^2}$ by w^ch was demonstrated in the first Coroll. compared with the 12 Cor. of the last Prop. but one. Take now $\sqrt{Dd^2 - Dd^2}$ to DA in y^e same proportion as before, these Lines were diminished & then proceeding y^e Operation, restore again Dd & Coroll. the Vel. in D will be increased in the same former magnitude. & by the foregoing Operation, that the Line DA is now diminished in y^e proportion $\sqrt{DA^2 - DP^2}$ will also be decreased in the same Proportion. For since DP is as DA, DP^2 will be as DA^2 & $DA^2 - DP^2$ as DA^2. & $\sqrt{DA^2 - DP^2}$ as DA. Therefore this Quantity $\sqrt{DA^2 - DP^2}$ or its equal DM. will still express the Velocity of y^e oscillating Body, in the Point D whatever be the Length of y^e Diameter AE.

Cor. 5. The Time wherein the Apellatory Body 110
passes over any Part AB of the whole AC
will be to the time of an Entire Apellation.
this: that Line, as the Arc AM is to y^e Semicircle AMC.

For drawing another Perpendicular PQ infinitely
near to MP. the time wherein the infinitely small
Space PQ is described will be as PQ is. as y^e
Space PQ directly, & as y^e velocity D M inversly.
Join D M & draw Mk equal & parallel to PQ
cutting MD in k. & the similar Triangles
DMP. & MMk will give Mk or PQ : MD :: MP :
MD. therefore $\frac{PQ \times MP}{MD}$ Therefore y^e time
wherein the Space PQ is passed over is as the
Arc MM. Therefore the Time in which the
Space AD is passed over is as y^e Arc AM.
therefore y^e time will be to y^e time of an
Entire Apellation as the Arc AM is to y^e
Semicircle AMC.

NB That $\frac{\sqrt{D}}{\sqrt{99}}$ will not differ much from $\frac{Dx}{99x}$ in Prob: 13. we shall prove after having premis'd the following Lemma—

Lemma—

If there be two Magnitudes, whose Difference is small in respect of the whole, the greater be increas'd & the less decreas'd as much, till the new diff: be double or triple the old ones. I say the new Quantities will be in a Duplicate or Triplicate Ratio of the former very nearly. Let the two Quantities be 101 & 100 raise 101 to $101+\frac{1}{2}$ & lower this to $100-\frac{1}{2}$ i.e. 99.

their new diff: may be double their diff: I say 2ly that $101+\frac{1}{2}$ will be to $100-\frac{1}{2}$, in a Dupl: Ratio of 101 to 100 within a Trifle for the Ratio of $101+\frac{1}{2}$ to $100-\frac{1}{2}$ is compos'd of these Ratios, viz: $101+\frac{1}{2}$ to $100+\frac{1}{2}$ :: $100+\frac{1}{2}$: $100-\frac{1}{2}$ this a little greater & that almost as much less as 2ly Ratio of 101 to 100. Again raise 101 to 102 & bring down 100 to 99. & 102 : 99 will be almost in a Trip: Ratio of 101 to 100. for 102 : 99 Contains in it 2 Ratios, of 102 : 101 :: 101 : 100 :: 100 : 99, 2ly but is something less, the last something 2ly. than the Ratio of 101 to 100

Cor: Where the Diff: is very small in regard to 2ly Magnitudes themselves, we need not change both but one only, on these remaining the same—

Hence if Nos 2 Quantities PR & QR whose Diff: is very small in respect to the whole Be changd to PR & DR whose Difference is double their Difference Diff: PD will be to PR in a Dup: Ratio of PR to QR—

Tab: 7. Page 110

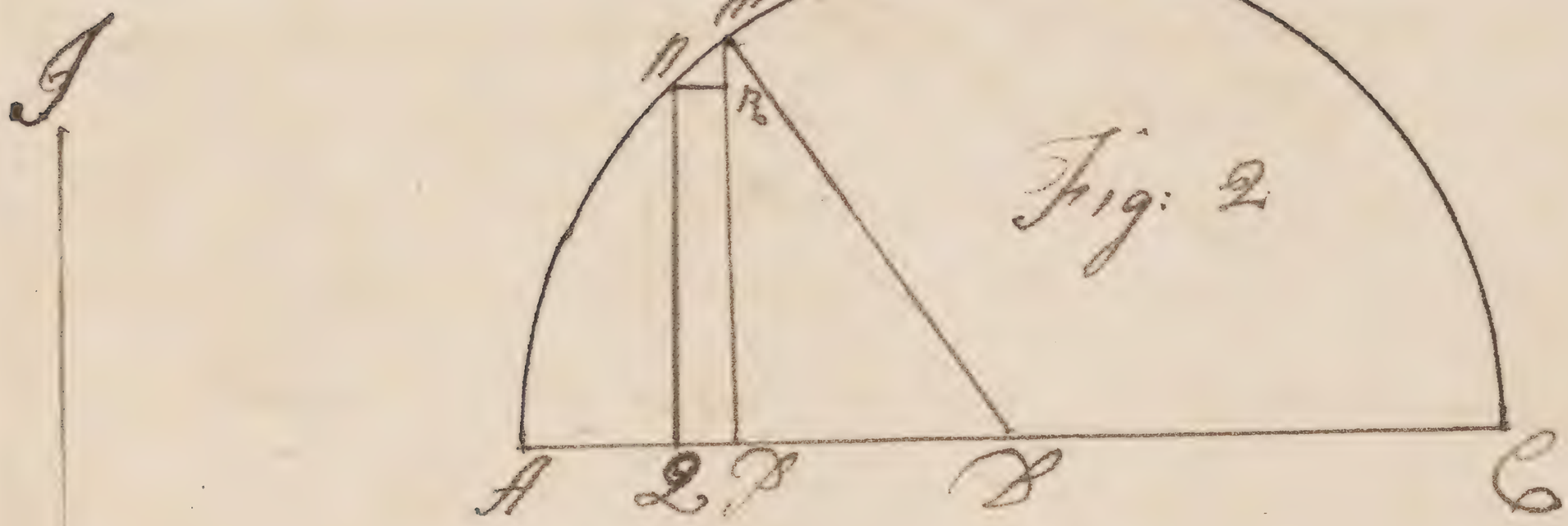


Fig: 2

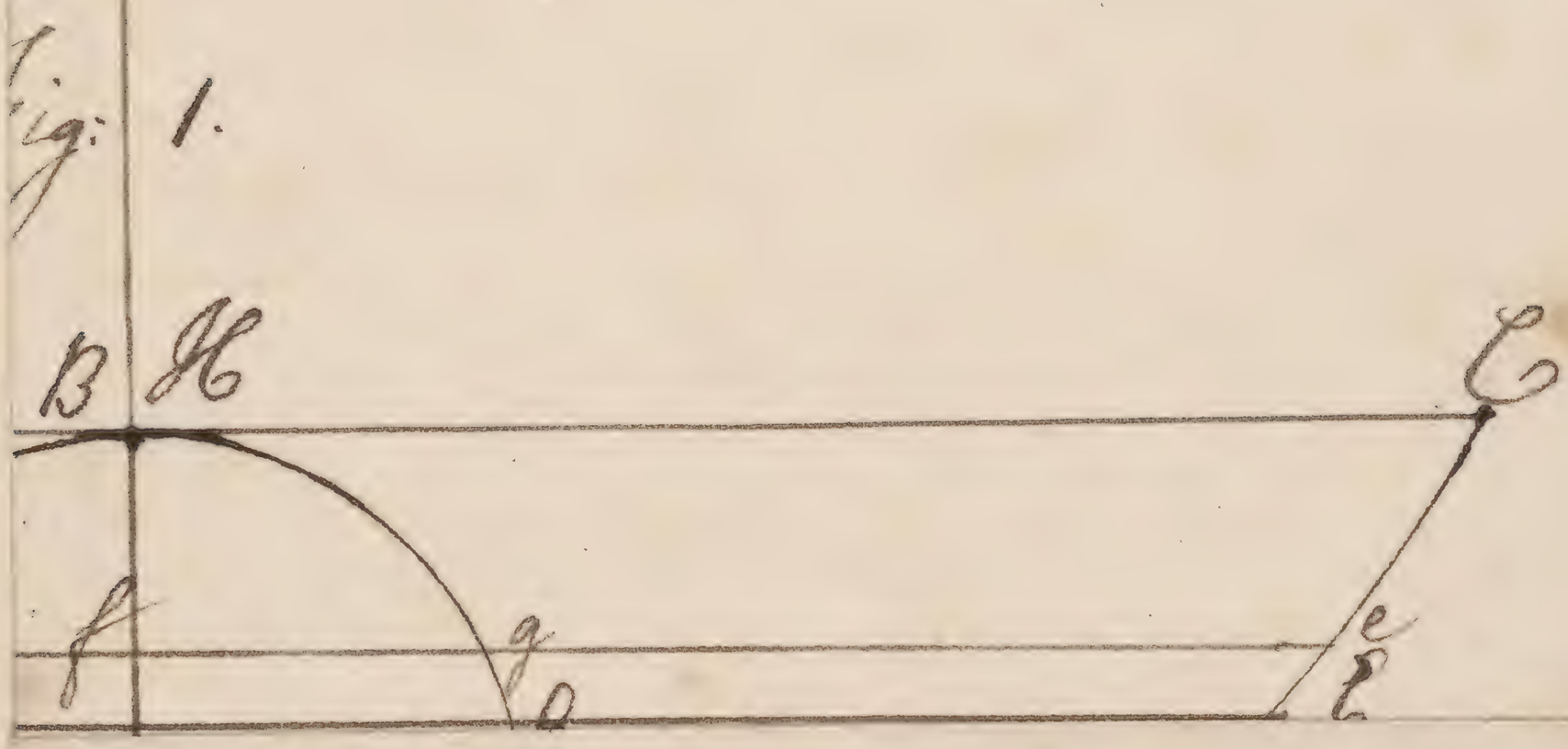
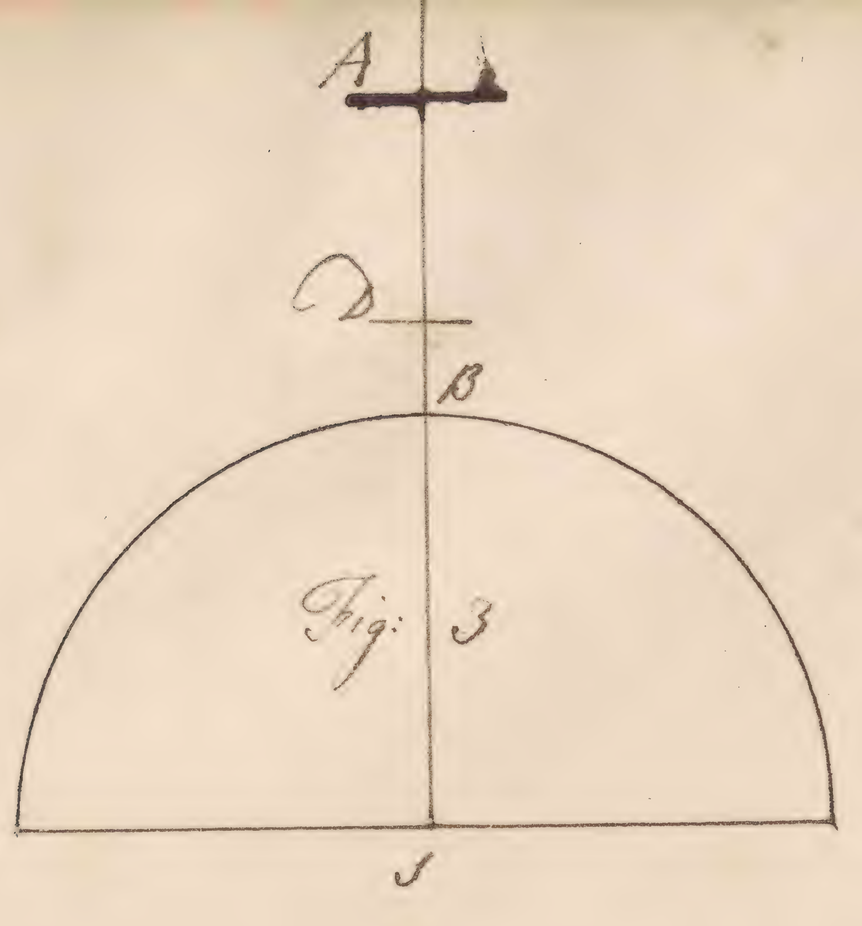


Fig: 1.



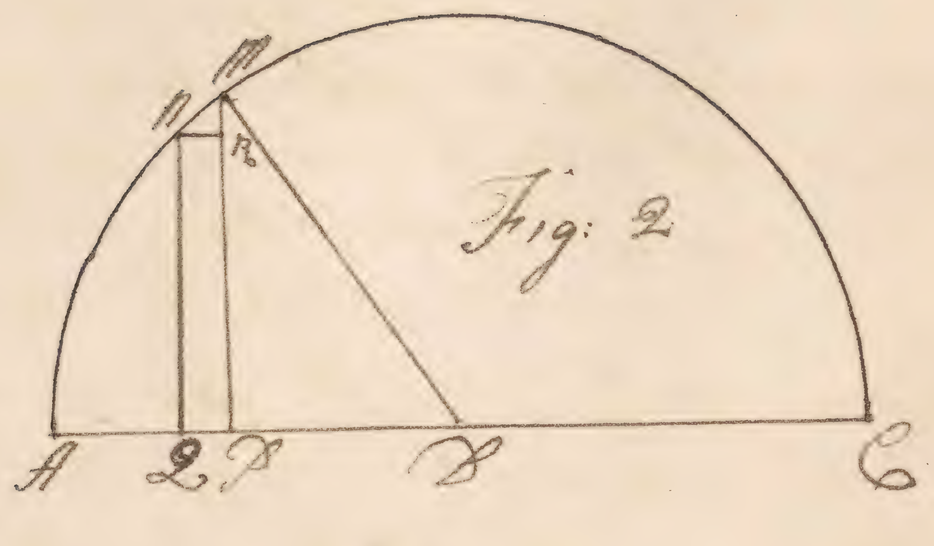
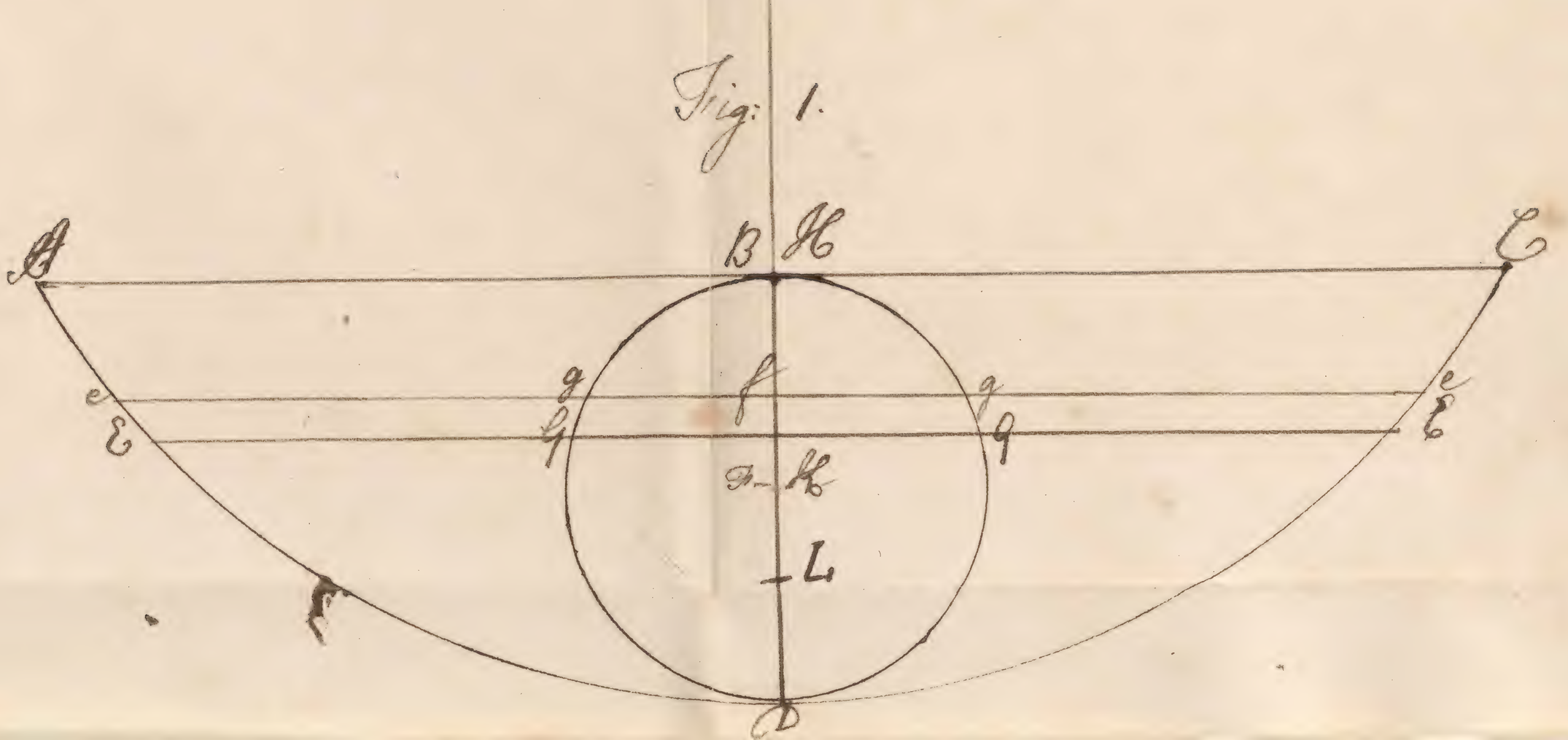


Fig: 1.



Of the Density of the Air —

Hypothesis

That the Density of ye Air is proportionable to the Compressing force whereby it is rarefied —

Tho' in Extreme Cases, this Hypothesis be attended with some difficulties / for it supposes ye Air capable of being both condensed & rarefied in infinitum, & whether some other absurdities / yet as far as any Experiments hitherto made can reach it is not found liable to any Exception ——— ——— ——— ——— ———

Lemma.

Tab 8
Fig: 2
Page 136

Let A be the Center of ye Earth. AB its semidiameter & the Height of an Homogeneous Atmosphere of the same weight w^th our own, and every where of ye same specific with common Air at ye Surface of ye Earth, & let AB be an infinitely smallest part of AB: I say then that then that the Density of our Air at ye Point B will be its Density at ye Point B. as AC to BC —

For supposing our Atmosphere to be annihilated & let the homogeneous Atmosphere above described be placed instead of it, & let the Line AC be considered as the Axis of a Column of this Air; then this Column is an Homogeneous Body the weight incumbent upon C will be to ye weight incumbent upon B. as AC to BC. But the weight incumbent upon C, from the former Atmosphere, was ye same as ye weight incumbent upon A from the latter. because both Atmospheres were supposed to have ye same weight. Moreover the weight of the Air included between A & B was the same in the former Case as it is in the latter, because both the Spaces AB, & the specific gravity of the Air included within it, were supposed in both Cases to be the same ———

but that the weight of the Air between B & A from the weight at A. & there will remain y^e weight at B in both Cases the same since y^e the weight upon A was to the weight incumbent upon B as AC to BC it follows from the Hypothesis already assumed y^t the Density at A. was to y^e Density at B as AC to BC

Corollary

Hence Immediately the Density at A will be to the Excess of the Density at A above the Density at B, as AC to AB. That is calling AC l, AB or BC x, and the Density at A or B v, we have v to $\dot{v}$ as l to $\dot{x}$, and therefore at the lth Distance $a\frac{v}{v} = \frac{\dot{x}}{l}$

Problem. 14.

Tab. 8 Fig 2 Page 136.

To Compare the Density of y^e Air at any given Altitude with y^e Density at the Surface of y^e Earth, upon a Supposition of Gravity in every where y^e same. Let S be the Center of the Earth as before, AB its Semidiameter, AB any given Altitude above y^e Surface A. Call AB, x the Density of the Air at B. v & call the Height of an Homogeneous Atmosphere of the same weight with one AC, and every where of y^e same Specifi. Grav. ty with that of Common Air at the Surface of y^e Earth, C. & let the Altitude AB also represent part of the Axis of a Column of Atmosphere, incumbent upon AC & supposing this Column to be resolved into an Infinite number of infinitely thin Lamina. Call the thickness of one of these Lamina, that viz. is contiguous to the Point B. $\dot{x}$ & its weight will be as $v\dot{x}$ i.e. as its Density and magnitude Compounded together. But the weight of every Lamina is the

Flexion of the Pressure or the Place where it exists. and as the Density is every where proportionable to the Pressure, the flexion of the Density will be proportionable to the flexion of the Pressure. i.e. i will be every where as $\frac{zi}{c}$ where fore $\frac{zi}{c}$ will be as zi & consequently as zi. But at 4° Dens. $\frac{zi}{c} = zi$ But the fluent of zi is $\frac{zi^2}{2c}$ whose value at the Point A is $\frac{AC^2}{2c}$ & at the Point B. $\frac{BC^2}{2c}$ & the Difference of these two values is $\frac{AB}{2c}$. Again the fluent of i is the Hyperbolic Log. of c. by what has been formerly Demonstrated. The Latter where of at the Point A is the Log. of 4° Density at A, & at the Point B. the Log. of Density at B. & the Difference of these two Values is the Log. of a Fraction, whose Num. is to the Denom. as the Density at A to 4° Density at B. therefore AB is the Hyperbolic Log. of such a Fraction, make, e, 2.7182818284590452353602874 = m, & $m \times \frac{AB}{c}$ will be Briggs Log. of the same Fraction. Therefore whenever the Altitude AB is given. find the fraction whose Log. is $m \times \frac{AB}{c}$ and the Density of the Air at A will be to the Density of the Air at B. as the Num. of that Fraction to the Denom. 2 & 1. —

N.B. Every whole number must be here conceived as a Fraction whose Denominator is Unity — —

* For $\frac{r}{s}$ be divided by the fraction $\frac{s}{t}$ the
Quotient will be $\frac{t}{r}$. Therefore if from the Log.
be subtracted the Log. of $\frac{s}{t}$ there will remain the Log.
of $\frac{t}{r}$ but the Log. of $\frac{r}{s}$ is $*$ & the Log. of $\frac{s}{t}$ is
$m \times \frac{ab}{c}$ & $m \times \frac{ab}{c}$ subtracted from $*$ leaves
$-m \times \frac{ab}{c}$ Therefore $-m \times \frac{ab}{c}$ is the Log. of $\frac{t}{r}$.

Cor. 1. If we call the Density of the Air at the Surface of the Earth, 1. the Density at any other Altitude will be expressed by a Fraction whose Log. is $= -m \times \frac{AB}{l}$ that is $m \times$ the Log. of the Fraction $\frac{x}{y}$. The Density at a mile be to the Density at B as x to y and so / to $\frac{1}{x}$ whose Log. is $= -m \times \frac{AB}{l}$.

Cor. 2. Whence since m & l are constant Nos. the Log. of the Density of the Air at B. will be as the Altitude AB.

Cor. 3. Whence it follows from the Nature of Log.s / if any number of Altitudes be taken in Arithmetical Progression, the Density of the Air at those Altitudes will be in a geometrical Progression / i. it will be Denominated in a continual Geometrical Proportion.

Cor. 4. If the Density at B be given the Altitude AB may be found thus. Let the Density at A be to the Density at B as x to 1 & let t be the Log. of the Fraction $\frac{x}{1}$ then will $t = -m \times \frac{AB}{l}$ & Conf. $AB = l \times \frac{t}{m}$

Scholium

The Specific gravity of well purged Mercury is to that of water as about $13\frac{2}{3}$ to 1, & the Specific gravity of water is to that of Air as about 870 to 1, when the Mercury in the Barometer stands at the Height of 30 Inches but these two Ratios together & the Specific gravity of mercury will be to that of Air when the Mercury in the Barometer stands at

30 Inches as 11890 to 1. Therefore the
Altitude of an Homogeneous Atmosphere of 4°
19 mc weight and 29 Inches Mercury with the Common Air at the
Surface of the Earth which in the foregoing
Computation we called, 1 mile be 35 6700
Inches, or 2992 Feet or 5, 6293 or
5, 631 English Miles. very nearly. Wherefore
if the Altitudes of AB be taken in Miles
& pts of Miles we shall have $\frac{m}{c}$ —
= 0, 0771429 & $m \times \frac{AB}{c} = AB \times 0,0771429$. —

Problem 15

To Compare the Density of the Air at any
given Distance with its Density at the Surface
of the Earth, upon a Supposition that the force
of Gravity is every where Reciprocally as the
Squares of the Distances from the Center of
the Earth. —

Solution.

Using the Notation of the foregoing Solution
the weight of an infinitely thin Lamina of Air
contiguous to the Point B, will now be as $\frac{Ux}{xx}$
because the force of gravity is as $\frac{1}{xx}$ therefore
i will be now as $\frac{Udx}{xx}$ & $\frac{di}{c}$ as $\frac{dx}{xx}$ or as $\frac{ff^2}{c} \times$
$\times \frac{-dx}{xx}$ but at the Point A, $\frac{i}{0} = \frac{x}{c} = \frac{ff^2}{c} \times \frac{c}{xx}$ —
but the Fluent of $\frac{dx}{xx}$ is $= \frac{1}{x}$ whose Value
at the Point A is $= \frac{1}{ff}$ and at the Point B
$= \frac{1}{fB}$. and the Difference of these two Values —
is $\frac{AB}{ff \times fB}$ Therefore $\frac{ff^2}{c} \times \frac{AB}{ff \times fB}$ or $\frac{ff}{fB} \times \frac{AB}{c}$

Cor: 1.ª For: AB: AH:: AB: AD by 3.º Construction
Therefore AB: AB − AH:: AB: AB − AD.
AB: AB :: AB: BD.

is the Hyperbolic Log: of a fraction whose Num: is to the Denom: as the Density at A to the Density at B. therefore $\frac{AA}{AB} \times m \times \frac{AB}{l}$ will be Briggs Log: of the same fraction — Dimin: the Altitude AB into the Altitude AD, by making as AB is to AC, so is AB to AD. Then shall have $\frac{AA}{AB} \times AB = AD$, & $\frac{AA}{AB} \times m \times \frac{AB}{l} = m \times \frac{AD}{l}$. Therefore if the Density at D be found by the method of the foregoing Problem upon a Supposition of the Uniform Gravity. the same will be the Density at B upon a like Hyp: of Gravity. ———

Cor. 1. The Altitude AD may be found a little more expeditiously by taking BA, BA, BD, continual Proportionals & then subtracting BD from BA ———

Cor 2. If AB. AC. & be taken continual Proportionals. & unto the equal Lines AB. AA :: AA. AE. we shall have dividendo AB. AC :: AA. AE. & alternately AB. AA :: AC. AE. But as AB. AA :: AC. AD by the Construction therefore AE = AD. —

Cor 3. If several Altitudes AB be taken in Musical Progression the Densities of the Air in those Places will be in Geometrical Progression; For Quantities are said to be in Musical-Progression, when their reciprocals are in Arithmetical Progression. Therefore if the several Altitudes AB be taken in Musical Progression; their reciprocals AC will be in Arithmetical Prog.— Therefore the several Altitudes AB will be in a Logarithmetical Progression, Therefore the Densities of the Air at their respective Altitudes AB will be in Geometrical Progression ——

Scholium —

To apply the numbers of the foregoing Problem, to this, the several Altitudes AA. AB. AD. in Miles and parts of a mile. in Order to ascertain the Semidiameter of the Earth is, about 3960 Miles.

Here it may not be amiss to take notice that (tho' our assumed Hypothesis (that the Density of the Air was proportionable to the compressing force) was strictly & universally true, Our Atmosphere must be inclosed in no less than an infinite Quantity of Air surrounding it to prevent its flying away, or which is the same thing, to preserve a finite Density at the surface of the Earth. for since AA. AB. AD. & so AB continued the less if AA be infinitely less than AB, AB will be infinitely less than AC, in which case the Circle described with C the Centre and AB will be equal to AC. Therefore AD will be finite when AB is infinite, & Consequently the Density of the Air at the infinite Altitude AB. will be finite & may be determined by y^e Solution of the Problem —

Ex. 6 Prob. 14 Cor. 1st

If the Density at the surface of the Earth be 1 & Log. of 1 & Dens. 1 at 1 Mile high. will be

$-1 \times \frac{m}{c} = -5 \times 0,0771429$ & $0,0771429$ subtra. [illegible]

from $0.$ $\frac{0\ 0\ 0\ 0\ 0000}{-1,9228571}$ & No. N° belonging to this Log. nig.

$-1,9228571$, is, $0,80725$. that is, the Density of 1 & Air at the Earth is to its Density at the height of 1 Miles as 1, to, 83725. or as 100000 to 83725. & in like manner if the Dens. the of the Earth Air at 1 & Earths surface be called 100000 are stated hence. the Densities affixed to 1 & Respective Altitudes as follows.—

From this Table it appears	Miles high.	Densities of Air—
that at the Height of 10 Miles	0	100000
the Air will be 250 times rarer	1	83725
1 & at the Earths surface—	2	70099
And at the Distance of 1	3	58691
semidiameter of 1 & Earth, its	4	49137
Density will be to the	5	41143
Density at the surface	6	34446
as 1. to 100000 & the	7	28841
Cyphers running on	8	24147
to 3 hundred & six—	9	20213
	10	16927
	40	82

of the Density of the Air at the Surface of the Earth be called s. the Log: of its Density at the Altitude AB will be $s - \frac{m}{n} \times AB$. But if the Density of the Air at y^e Surface of the Earth, be call'd 100000 then its Density at the Altitude AB ought to be represented by a number 100000 as greater as y^e former. & therefore its Log will be $5 - \frac{m}{n} \times AB$. compute therefore the L^o of this Log. & it will represent the Density at AB upon supposition that the Density at A is 100000

Thus if AB be 1 Mile —— 5, 0000000

$\frac{l}{n} \times AB$ —— —— —— 0, 0771429

Log: of the Density = 4, 9228571

& the Density it self —— 83725

And by this means a Table may very easily be made

Log to Prob. 14. Cor. 4th

Since $AB = \frac{l}{m} \times t$ Log. AB will be $= \log. \frac{l}{m} + \log t$ but $\log. \frac{l}{m}$
$= 1,1127035$ for $\frac{l}{m} = 1$ Divided by $\frac{m}{e} = 0,0771429$
whose log is from the log. of 1 are to be subtracted & the log. of
$0,0771429$ Hence we remain $\begin{array}{r} 0.0000000 \\ -2.0912965 \\ \hline -1,1127035 \end{array}$ the log of $\frac{l}{m}$.

Now let the Altitude AB be required to which the air at B may be half as dense as at the surface of our Earth. Then will $t = \text{log. of } \frac{2}{0}$ & the log. of t will be the log. of the log. $= -1,4780698$ & $\log. \frac{l}{m} + \log. t$ will be
$\begin{array}{r} 1,1127035 \\ -1,4780698 \\ \hline 0,5913193 \end{array}$ & the No. belonging to this log. is 3,9022. at which Altitude the air is $\frac{1}{2}$ as dense as at the surface of the Earth. ———

In the manner we shall find the air to be $\frac{1}{3}$ as dense at the height of 6,1849 Miles, $\frac{1}{4}$ as dense at 7,2048 &c. ——

$\left\{ \begin{array}{l} \text{Log } 3 = 0,4771213 \\ \text{Log. of Log. } 3 = -1,6786216 \\ \text{Log. of } \frac{l}{m} = 1,1127035 \\ \text{Log of } AB = \underline{0,7913311} \end{array} \right.$ whose Natural No. is 6,1849
Miles as above ———

Exc. to Prob. 15. Cor. 1st

Let it be required to find at the Height 1 Mile the Density of y^e Air in y^e latter Hyp: Now we have 3960 = BF. : r = AB :: r = AB : 000252 = BD. Now AB − BD = DE & $\frac{DE}{1-000252} \times \frac{m}{e}$ will be Log: sought i.e. 1 × 0,77143 − 0,77143 × 000252 i.e. 000194 subtracted from 0,77143 = 0,771236 the num: belonging to y^e Log: when subtracted as 4 figures will be 5,37129 but before at this Altitude it was 5,37125. so that y^e Density is computed from both these Hyp^s the Hyp^s differs only in the 5. Place.

Now to find y^e Density at 10 Miles high: since BF : BA :: AF : BD we shall have $\frac{BF^2}{AF}$ or since AF differs little from AB $\frac{BF^2}{AB} = BD$ & since AB is a constant Qu^ty BF^2 will be as BD × $\frac{m}{e}$^10 & since we shall have the Log: of the Altitude 1 Mile: BD × $\frac{m}{e}$ = 0,000194 :: the Log: of y^e Altitude 10 Miles : 00194 = BD × $\frac{10m}{e}$ at y^e Altitude 10 Miles. but AB − BD × $\frac{m}{e}$ = $\frac{ABm}{e} - \frac{BDm}{e}$ will be the Log: sought. Therefore we are to subtract 00194 from $\frac{m}{e}$ × 10 = 771430 & there will remain 769236, which subtracted from × leaves 1,23051. & y^e N^o belonging to y^e Log: is 17012 that is the Density upon the Latter Hyp^s at the Surface of the Earth, is to the Density at 10 Miles high, as 1000000 to 17012 upon the former Hyp^s: it was found to be 100000 to 16912, therefore the Air is more Dense, rarified from the latter Hyp^s y^n the former, at the assigned heights in the proportion of 17012 : to 16912 which is nearly in a Ratio of Equality, therefore at all Altitudes wherein it is impossible to make Experiments (such as in measuring the Heights of Mountains by the Barometer) the Hyp^s of Uniform gravity will be exact enough

Lemma 1st

Let 1 be the space through w.ch a heavy body descends in a second of time, arguing a Col.y w.ch would carry it thro' 2s. uniformly in y.e same time. & let 2 be y.e space described in a sec.d of time by a globe, whose diameter is 1 being made to move uniformly, in an uniform Fluid. Lastly Let e be an accelerating force, w.ch acting uniformly upon any Body, will generate a motion equal to that of the globe in y.e same time, as y.e wherein the globe describes $\frac{2}{3}$ of its own diameters. I say then that the force with which the globe suffers from the fluid medium, as the Density of the globe is to the Density of y.e fluid. This is y.e 98.th Coroll. of y.e 2.d Book. Prime. It is demonstrated, by y.e help of y.e 2 foregoing ones.

Lemma: 2d
a Problem

To find the Resistance of a given globe being made to move with a given Velocity in a fluid medium of a given Density.

Using the Notation of y.e foregoing Lemma since the globe by its uniform motion describes the Space 2 in 1" of time, it will describe $\frac{8}{3}$ in the time $\frac{4}{30}$. Let a be the weight of the globe, in vacuo, then the Col.y generated

by the ℊ globe whose is a, in 1′ of time, will be
to the Cel: generated by the face e, in ℊ same
ℊ globe and in ℊ same time, as a. b. c. Again
the Cel: generated by the face d, in ℊ ℊ globe, in 1″ of
time, will be to ℊ Cel: generated by the same
face e in ℊ same ℊ globe in the time $\frac{ℊ d}{3c}$ as 1′ is
to $\frac{ℊ d}{3c}$ or as 3c to ℊ d. But the 2 plates
together (being) the plate of a globe, & the plate
of 3c to ℊ d. Are called ℊ Cel: generated
in the globe by the faces a in 1′ of time, to the
Cel: generated in ℊ same globe by ℊ face e, in
ℊ same time, w.ch is $\frac{ℊ d}{3c}$ as 3ac is to ℊ de
But the former Cel: is that whereby the face
2d may be described uniformly in 1″ of time,
& the latter Cel: is by help whereof the ℊ globe
whereby 2c is described, in ℊ same time. Ergo
the former Cel: is to the latter, as 2d to 2c
or as d to c. Ergo: d : c :: 3ac : ℊde ——
Therefore $e = \frac{ℊ ace}{ℊ d}$ i.e. the face is equal
to the weight of the ℊ globe, in to the plate
$\frac{3ce}{ℊ d}$. But by the foregoing Lemma, ℊ face
is to ℊ Resistance the ℊ globe meets with, from
the fluid, as the Density of the ℊ globe
is to the Density of the fluid. or as the
weight of the ℊ globe, is to that weight of
so much of the fluid as, is equal in Bulk
to the globe. Therefore the Resistance the
globe meets with is equal to the weight
of so much of the fluid, as answers in Bulk

to the Globe x the Fraction $\frac{3cc}{4d}$ i.e. of G. be the Relative weight of the Globe in the fluid the Resistance sought will be $a-b \times \frac{3cc}{4ds}$.

Cor: The Resistance the Globe meets with from the fluid will be as the Q. of Diameter, as the fluid altogether. And the Quantity $\frac{3}{4}$ being Constant, the Resistance will be as $a-b \times cc$ a Let m be the Density of the fluid medium, then will $a-b$, or the Weight of the moved fluid as is equal in Bulk to the Globe be as md^3 i.e. as the Density and magnitude together. Therefore the Resistance will be as $md^3 \times \frac{c^2}{d}$ i.e. as md^2c^2.

Lemma: 3? Problem

To find the greatest Vel: a given Globe may acquire, by the relative weight in descending in a fluid of a given Density.

This is nothing else but to find what Velocity a Globe ought to move with to meet with a resistance equal to its own relative weight in the fluid. For it is certain, that when so happens, the Globe must be incapable of further Acceleration. Therefore supposing all things as in the two foregoing Lemmas. Let c be the Space described in a second of time by the Vel. sought, and the Resistance will be $a-b \times \frac{3cc}{4ds}$ as above. Make the Resistance $= b$ the Relative weight of the Body, & we shall have $\frac{3cc}{4ds} = m \times a-b$

In the same fluid the Sp. of the greatest Part will be as the Diameter of the Globe, for in this Case $\frac{b}{a-b}$ is a given Qty so also is $\frac{16}{3}$ Ds ——

That $\frac{b}{a-b}$ is a constant Qty will appear, because in the same fluid. a is to a − b. that is the weight of the Globe in vacuo is to the weight of so much of the fluid as is equal in Bulk to the Globe, in a given Ratio. viz. as the Density of the Globe to the Density of the fluid. whence deriving. b : a − b :: in a given Ratio. Ergo. $\frac{b}{a-b}$ is a Constant Quantity ——

& $2c = 2m \times \frac{\sqrt{b}}{a-b}$. Therefore the Vel. the Globe can acquire will be such, as will carry it uniformly, tho' the Space, $2m \times \frac{\sqrt{b}}{a-b}$ in a second of Time. Q.E.D.

Cor. In the same Fluid the Vel. of y^e greatest Vel. will be as $\frac{b}{da}$ for in a Cube $a-b$ will be as $\frac{b}{da}$ and therefore the Vel. $\frac{b}{a-b} \times \frac{16}{3}$ do will be as $\frac{b}{da}$.

Lemma: 4.th Problem

To find how long & how far a given Globe ought to descend by its comparative weight in a medium of a given Density, but with any Resistance, to acquire the greatest Velocity it is capable of in descending w.th the same weight, & in y^e same medium with Resistance.

Let f be the Space, & g the time required, then will the Vel. of the Globe acquired in falling tho' the Space f, by the weight a, be to y^e Vel. acquired in falling tho' the Space f by the weight b, as the Vel. is to the Vel. But the Vel. of the former Vel. is to the Vel. of the latter, as $\frac{f}{a-b} \times \frac{16}{3}$ do. whence $f = \frac{a}{a-b} \times \frac{4}{3}$ d) i.e. the Space is to $\frac{4}{3}$ of the Diameter of the Globe as the Density of the Globe is to the Density of the Fluid) — Q.E.D.

Again. Since the Spaces thro' w.ch any Body is uniformly accelerated is as the accelerating force & the Sq. of the time of its Generation

we shall have the q. s to the space f. as $a \times 1.2$
(1.2. as a multiple × the square of the time)
to $b \times gg$ whence gg will be found equall
to $\frac{a}{b} \times$ f. & $\sqrt{\frac{af}{bs}}$ 1.2. the time sought will
be to the time of 1". in a subdupl. Ratio of
a f to b s. Q E D +

Cor. 1. If a given Ball was to move in a
right line in a fluid medium of given density,
and its find Resistance was to encrease along
the same, like it had gone down the middle
of the Ball, the space the Ball will go
through, will be always the same.
For it will always be as the square of the first
Vel. directly, and as the Resistance i.e. the q.
of the same Vel. inversly. Therefore if this
Space be known in any one particular instance
it will be known in all others. But in the
Case where the q. of the first Vel. is $\frac{b}{a} \times \frac{16}{3}$ ds.
this Space is known, to be the Space
described by the Centripetal force of the 1st Resistance
will in y^e Case have the same effect as
a like Continuation of the relative motion 6.
Therefore in all other Cases the Space
above mentioned will be f. — —

Cor 2. The time wherein the above mentioned
Effect will be produced, will be that wherein
the Space 2 f. will be uniformly
described, by the first Velocity —

Lemma 5th

The fluent of a Fraction whose numerator is the Fluxion of the Denom. is the Hyp. Log. of the Denom. But it has been already shown, that the fluent of $\frac{\dot{z}}{z}$ is the Hyp. Log. of z.

Cor. 1. Therefore the Fluent of this Fraction $\frac{2v\dot{v}}{mm-vv}$ is the Hyp. Log. of $mm-vv$. i.e. it is the Log. of $\frac{1}{mm-vv}$.

For $\frac{2v\dot{v}}{mm-vv} = \frac{-2v\dot{v}}{vv-mm} \times -1$ whose Fluent is the Hyp. Log. of $vv-mm \times -1$ i.e. the Hyp. Log. of $mm-vv$ a the Hyp. Log. of $mm-vv$ inverted from $*$ i.e. the Hyp. Log. of $\frac{1}{mm-vv}$

Cor. 2. & the fluent of a Fraction $\frac{2m\dot{v}}{mm-vv}$ is the Hyp. Log. of $\frac{m+v}{m-v}$. for $\frac{2m\dot{v}}{mm-vv} = \frac{\dot{v}}{m+v} \times \frac{\dot{v}}{m-v}$ & therefore the Fluent will be the Hyp. Log. of $m+v$ minus the Hyp. Log. of $m-v$. i.e. it will be the Hyp. Log. of the Fraction. $\frac{m+v}{m+v}$ —

Problem. 16th

Let a given ball passing in a strait line thro' a fluid medium of a given density, be resisted every where in Proportion to y^n of its velocity. Its velocity having given the Vel. of y^e Ball at its first entring, to find naturally the motion, it will loose, but also the space thro' which it will have passed in any given time assumed. ——

Let d. be the diameter of y^e globe & let y^e space which y^e globe in describing by the first Resistance of y^e medium shall be the Denom^r of the first Vel. & let m be the first Vel. & let the time wherein the space z may be uniformly described by the first Vel. m.

Then will $\frac{z}{m}$ be the time wherein the first Resistance $\frac{1}{3}d^3$ will uniformly continue to destroy y^e first motion of the Ball, by the Pr. to y^e Lemma. Let x be the space described by y^e Ball in y^e indeterminate Time t from its first entring and let v be the Vel. & $\frac{1}{3}d^3$ y^e Resistance at y^e End of the Time t. Then since the Vel. generated or destroy^d by any uniformly accelerating or retarding force is as the force multiply^d into the Time of its Action, we shall have y^e Vel. m. to the Vel. v as $\frac{1}{3}d^3x$ to the product of y^e time & force whereby the former Vel. may be destroy^d to $\frac{1}{3}d^3x$ & the Product of the time & force whereby the latter Vel. is destroy^d $\frac{1}{3}d^3t$ :: mm. ov by the Hyp: Therefore $\frac{1}{3}d^3t$ :: $\frac{1}{3}d^3$ mv :: vt. $\frac{1}{3}d^3$ mv :: vt. Therefore $t = \frac{\frac{1}{3}d^3x}{v} = \frac{z}{m}$

But the fluent of $\dot{t}$ is t & the fluent of $mg \times \frac{\dot{v}}{v}$ is $mg \times \frac{1}{v}$ whose value at 4° beginning of the time t is $mg \times \frac{1}{m}$ & at 4° End of 4° time t, is $mg \times \frac{1}{v}$ & the Diference is $mg \times \frac{m-v}{mv}$ $= \frac{gm-gv}{v}$ hg. $t = \frac{gm-gv}{v}$ & consequently $\frac{mv}{gm-gv}$ as the Velocity at 4° End of 4° time t will be $\frac{gm}{g+t}$ & therefore the Velocity destroyed by the Resistance in the time t will be such a part of v as gm is expressed by $\frac{gt}{gm+gt}$.

Now to find the Space we have 4° following Theor. (VI.) As the Space s is to 4° Space v so is mv the product of the time & Velocity the former Space may be uniformly described by & the Product of 4° time & Velocity the latter Space may be described. Therefore $s = \frac{1}{2}f \times vt$ Indeed for t substitute its value $\frac{mv}{mg \times \frac{v}{v}}$ we shall have $v = 2f \times \frac{v}{v}$.

But the fluent of $\dot{v}$ is v & the fluent of $\frac{\dot{v}}{v}$ is the Hyp: Log: of v. when at 4° beginning of the time t & is the Hyp: Log. of gm. and at the End of the time t is the Hyp: Log. of v & the Diff: is the Hyp: Log. of $\frac{gm}{v}$ Therefore s is the Hyp: Log. of $\frac{gm}{v} \times 2f$. instead of v substitute $\frac{gm}{g+t}$ we have found & we shall have $s =$ the Hyp: Log. of $\frac{g+t}{g} \times 2f$ &c. Now as the Ball in the time T by its 1st Vel: M would describe uniformly, the Space $2f$, it will in the time t by the same Vel: M: describe the Space $2f \times \frac{t}{T}$ Therefore the Space

actually described by the Ball in the time
t with Resistance, will be to the space it w?
have described in the same time, had it moved
all along with the first uniform velocity
Resistance, as the Hyperbolic Log. of $\frac{g+t}{g}$
is to the Ratio $\frac{t}{g}$. Make $m = 4349$
$4819\,03$, & we shall have the Hyp. Log
of $\frac{g+t}{g} = b$ Briggs Log. of 4° same $\times$
$\times 2dm \cdot 1.8. \times 2,3025850\,93$. Therefore
the Space described, in the same time, with
Resistance is to the Space that would have
been described in the same time without
Resistance. as the Com. Log. of $\frac{g+t}{g} \times 2d \times$
$\times 2,3025850\,93$ is to the Ratio $\frac{t}{g}$.
See Newt. Princip. Edit 3? Page: 355.
Scholium.

In the foregoing Solution, the Space was
found equal to the Hyp. Log. of $\frac{g+t}{g} \times d \times 2f$.
or. is equal to Briggs Log. of 4° same Ratio, mul=
=tiply'd $\times \frac{2f}{m}$. $1.8. t = g$ & let the Density
of the Fluid the Ball moves in be equal to
that of the Ball, then will the Log. of $\frac{g+t}{g}$
be. the Log. of $2 = 30103$ & the Space
will be equal to $\frac{g}{3}$ & $\frac{2f}{m} = 6,14029$.
Therefore the Log. of $\frac{g+t}{g} \times \frac{2f}{m} = 6$, the
Space will be $1,848\,46$ &c. w^ch is somewhat
less than $2d$. Again. the motion lost w^ch
was $\frac{tm}{2f+t}$ will in this Case be $\frac{1}{2}m$. Therefore
a Ball projected in a Fluid of 4° same
Density with itself will loose $\frac{1}{2}$ its motion before
it has described a Space equal to 2 of its own Diameters.

Let a given Ball be supposed to fall from rest in a fluid of a given density being every where retarded by the excess of its compressive weight above the resistance of the fluid, its regula to Origin, the Space thro' w.ch it will fall, & the Velocity it will acquire in any given Time Let s be the Space & v the Velocity acquired in any indeterminate time t. & let Unity or 1, represent, both the Volume & weight of the Ball & the greatest Velocity it can acquire in descending thro' the fluid. Also let r represent the Resistance of the fluid answering to the Vel.y v. & r — vv will be the moving forces whereby the motion of the Ball will be constantly accelerated. Put f the given Space described by the Time of Descent necessary to be made for the same Body, to acquire by its Relative weight the same vel.y v. in a medium without Resistance. Then since the Vel.y is generated in a Time as $\frac{v}{1-vv}$ by the force 1. & since the Vel.y is generated in the time t by the force $1-vv$ we shall have $1 : \frac{v}{1-vv} :: 1 \times g : 1-vv \times \dot{p}$ whence $2\dot{p} = \frac{2\dot{v}}{1-vv}$ But the fluent of $2\dot{p}$ is $2p$ & the fluent of $\frac{2\dot{v}}{1-vv}$ is the Hyp: Log: of the Reaction $\frac{1+v}{1-v}$ (by 2.d Cor. 5. Lemma.) which at the beginning of the fall is nothing Therefore $2p$ is the Hyp: Log: of $\frac{1+v}{1-v}$ & of consequence v or $\frac{dx}{dt} = \frac{\text{Hyp: Log: of } \frac{1+v}{1-v}}{2p}$

Now $\frac{2\pi i}{9}$ will be the common log. of $\frac{1+v}{1-v}$ $= n$. Hence v will be equal to $\frac{e^n - 1}{e^n + 1}$. i.e. the Vel. v acquired in the time t will be $\frac{e^n - 1}{e^n + 1}$ of the greatest Vel. a as is expressed by the Fraction $\frac{n-1}{n+1}$ of unity. Something the Vel. v will be to it's greatest Vel. as $n-1$ to $n+1$. Q.E.D.

Again. since the Space s can be described by the Vel. 1 in the time g, & since the Space s is described by the Vel. v in the time t. we shall have $s : v :: 1 : gv$ whence $s = tv \times \frac{2\pi i}{g}$ instead of $\frac{2\pi i}{g}$ put its value above found, viz $\frac{2i}{1-v^2}$ & we shall have $s = t \times \frac{2vi}{1-v^2}$ but the Fluent of $\frac{2vi}{1-v^2}$ is the Hyp. Log. of $\frac{1}{1-v}$ by Cor. 1 Lem. 5. I have here the Space s is the Hyp. Log. of $1-v \times$ (or its Augm.) Log. of the same Fraction, multiplied $\times t$. But v has already been found equal to $\frac{n-1}{n+1}$ & $\log \frac{1}{1-v}$ is equal to $\frac{nn+2n+1}{2n}$. Therefore now we may say that the Space s is the Log. of $\frac{nn+2n+1}{nn}$ multiplied $\times \frac{t}{n}$. But this fraction $\frac{nn+2n+1}{nn}$ is the Product of these two others whereof one is $\frac{n+1}{n}$ & the other $\frac{n}{1}$. Let the Log. of $\frac{n+1}{n}$ then will $2l$ be the Log. of $\frac{n+1}{n}$. Let k be the Log. of $\frac{n}{1}$, & since the Log. of n was before found

There are three Terms expressing the Difference made in the Time of the Last of w^ch 4. Coasts may be neglected according to Mr Isaacs. if the Fluid only be made Deep enough; w^ch may thus be made out ——————

In the 1st Experim^t p was to g as 2 to 7 when 2 p = 54 & 2 p^m = 54 × 0, 43. or 23. nearly, & the Log: w^ch the number n united was something g^r than 23. & therefore the number, n than Unity w^ch 23 Cyphers. added to it. But l is 4^o Log: of this Number. $n + \frac{1}{n}$ & therefore as the Log: l, we must diminish, 434 &c by a number g^r than Unity w^ch 23 Cyphers. after it. 1 & f. in this Case, the number becomes a mere cypher, & of Course 4. Coasts of Holland in this Experim^t. the Third Term may safely be supposed even

equal to $\frac{2fm}{g}$ the Log: of $\frac{n}{n}$ viz the Log: of 1 $\frac{2fm}{g} - \frac{k}{g}$. but the first two Log: together
Were shall have the Log: of the Fraction $\frac{nn+2n+1}{nn}$
$= \frac{2fm}{g} - \frac{k}{g} + 2l$. Therefore the Space $= \frac{2fm}{g} -$
$- \frac{k}{g} 42l \times \frac{f}{m} = \frac{2pf}{g} - \frac{k}{m} \times f + \frac{2}{m} l \times f = \frac{2pf}{g} -$
$- 4386294361 f + 4,6051701860 \frac{f}{g}$
See Newt: Princ: Lib: 2. Prop: 40.

Scholium —

By various Experiments made upon the Resistance of Bodies, both in water & Air, this Theory, of Resistance is not only confirmed sufficiently, but it is also further demonstrated, that in those Fluids all that Resistance which arises from the viscosity of the Fluid, or other, are compleatly removed.

See the Schol: to the 40. Prop: of 2. Book of Princ:

Tab. 4. D. Page. 136

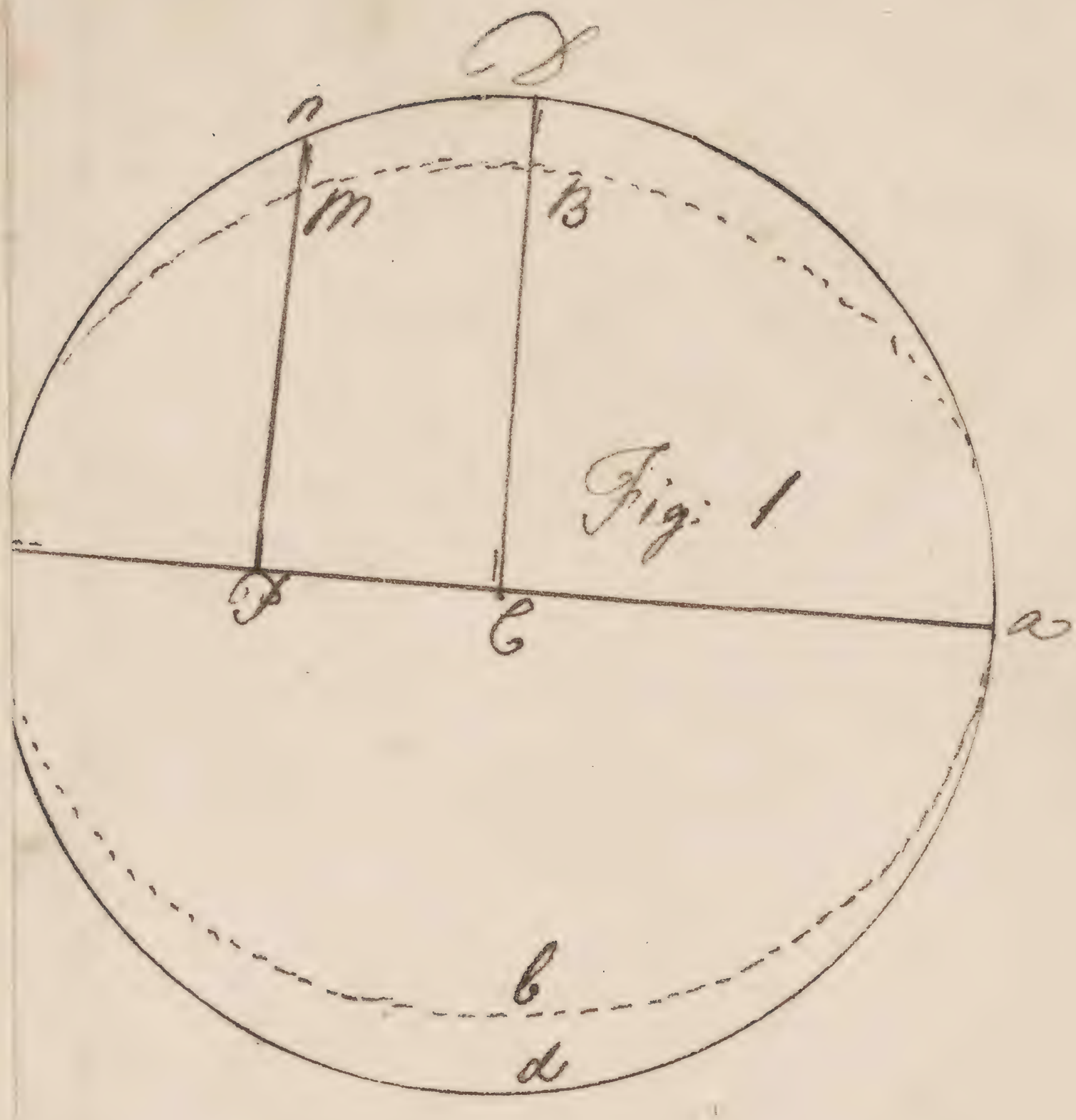


Fig: 1

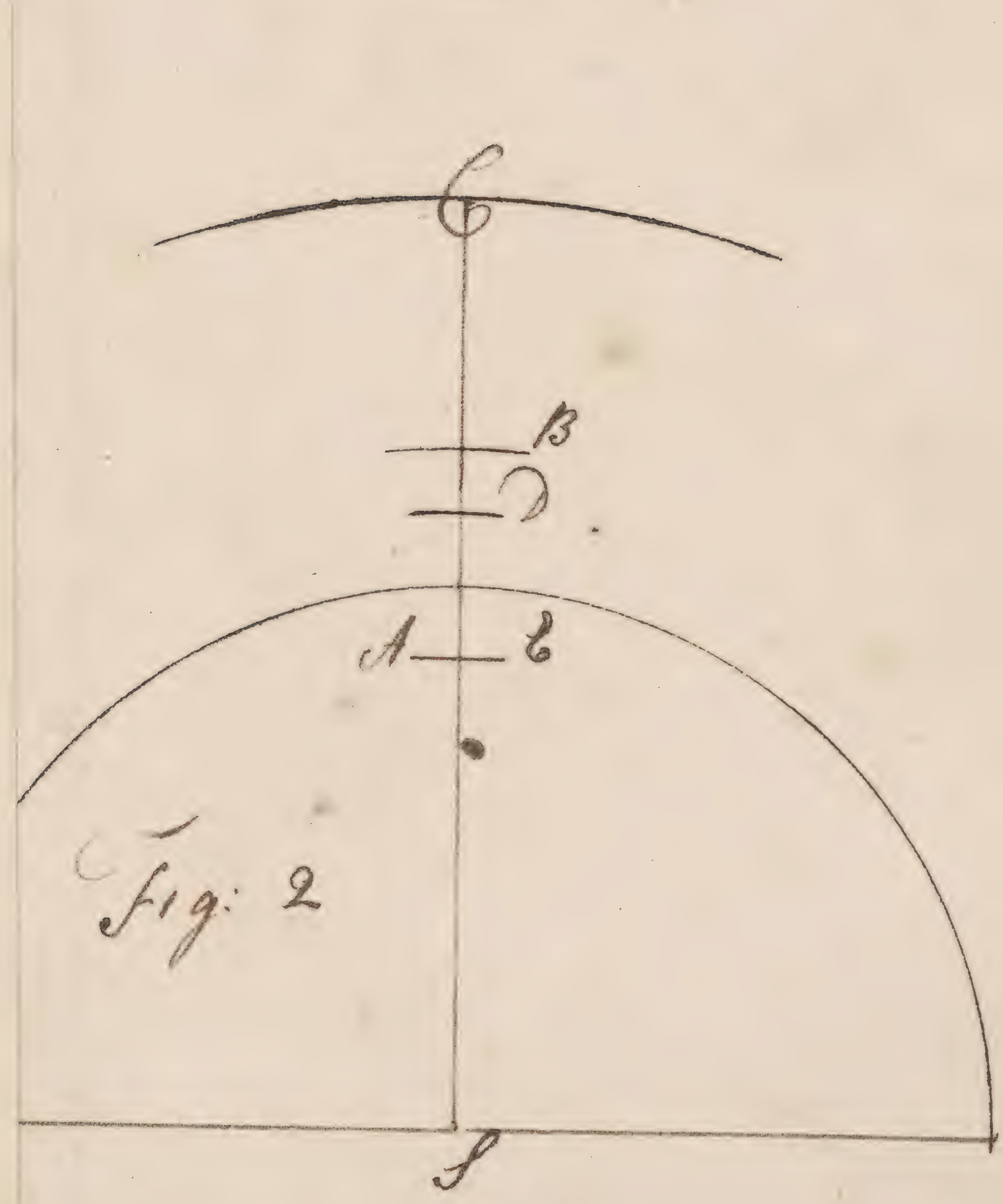


Fig: 2

Tab. 8. Page. 136

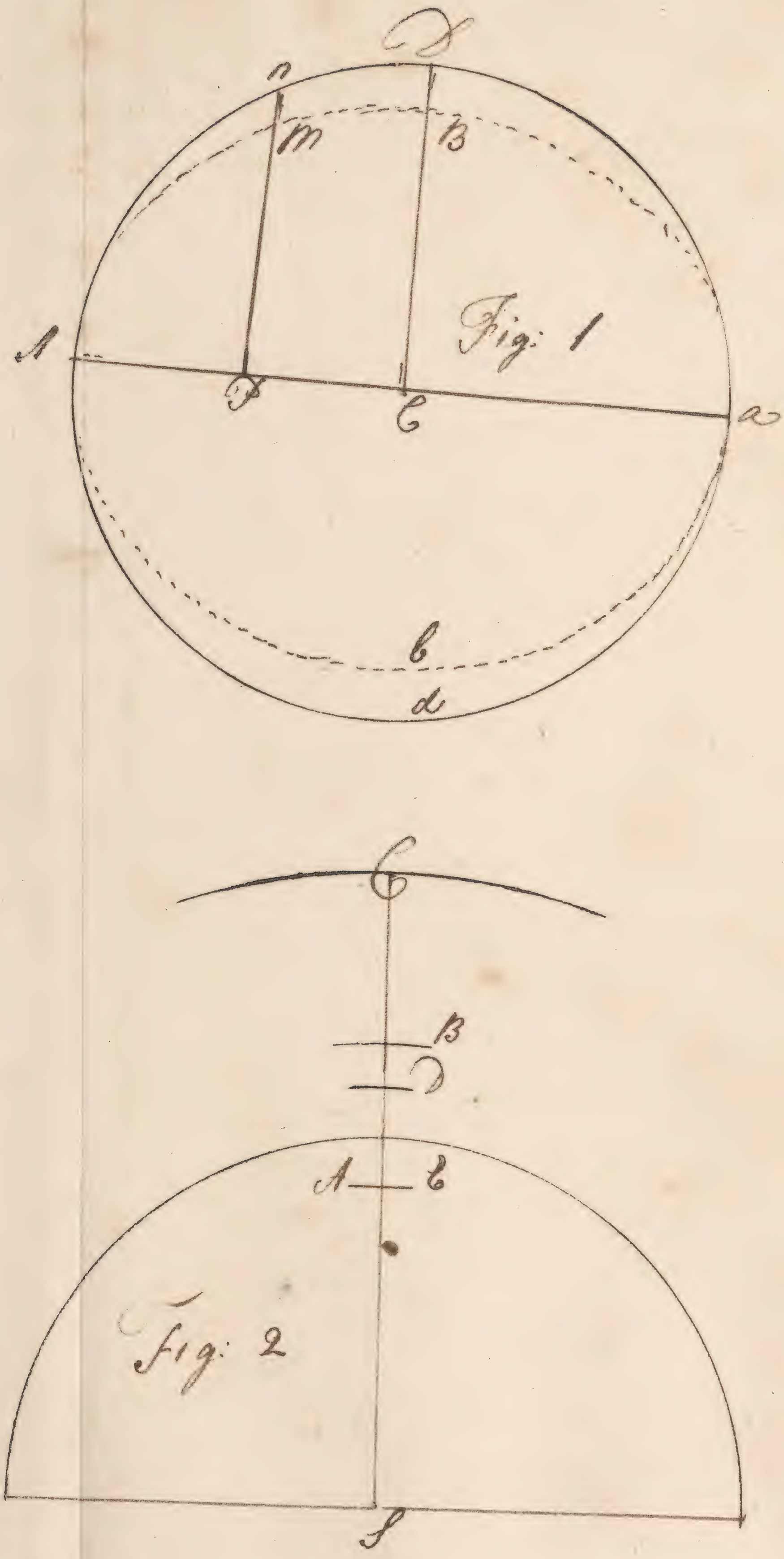


Fig. 2

of the Motion of a probable Spheroid upon
Particles of matter, placed any where, in its
Axis produced, necessary for understanding
the figure of the Earth —

Prop. 1st Theorem —

Pg 1
Pl 1
P. 144

Let L H (x) be a line given in position & let P Q (y)
be supposed to move perpendicular to L H & let
$a - 2bx - cxx = yy$, the Quantities a, b & c being constant
I say then that the Point Q, will describe 4° Sections
of a Circle whose Center is somewhere
in the line L H or, in 4° Line P L produced —

Demonstration —

Produce the line P L out then. & let D, or y^t. D
may be $= \frac{b}{c}$ & call D H (3), then will L H or $x = 3 - \frac{b}{c}$
$b - 2bx = -2b3 + \frac{2bb}{c}$ & $-cxx = -c3^2 + 2b3 - \frac{bb}{c} =$
$= a + \frac{bb}{c} - c3^2 = \frac{bb + ac}{c} - c3^2$. Therefore $yy = \frac{bb}{cc}$
$\frac{bb+ac}{cc} - 33$. Make $\frac{bb+ac}{cc} = rr$, & we shall have
$yy = rr - 33$ w^h. is the nature of a Circle whose
Center is D. & Radius $= r$ —

Prop. 2? Theorem —

Pg
Pl 2
P. 144

Let ABab, be an Ellipsis, & the greater Axis
A a the lesser Axis, C the Center, & if 4° Right Lines
Dmit any where in the lesser Axis Bb produced
from B. D a Line making the Ellipsis any where
in the Point D, & if a Line drawn thro' D.
perpendicular, to 4° greater Axis A a in E. & let
belong to E & be variable, that the Line
D may always have the same ratio to E F.
that F C hath to C B. I say the x^t. 4^e Locus
of the Point F, will be in the Circumference
of a Circle whose Center is somewhere
D or in x^t. Line produced, from. C
Dem: Call C B (c), E B (b), E F. (x), F P. (y), C P. (z)

& we shall have $Db = d \mp x$ & $Db^2 = d^2 \pm$
$\pm 2dx + x^2$. Moreover by reason of the
nature of the Ellipse $bD + bE = 2a$ &
$bE = 2a - bD$. whose DE^2 or $Db^2 + bE^2 = dd + tt \mp 2dx + xx$
$- \frac{ttxx}{cc}$ or $\frac{ccdd + ttxx}{cc} = - \frac{2attxx}{cc}$ Now DE^2
or $bE^2 \mp \frac{2attxx}{cc}$ $= dd + tt \mp 2dx - \frac{2attxx}{cc}$. Now we
by the foregoing Proposition the Locus of the
Point E, in the Circumference of a Circle whose
Centre is in y° Line Db, or any Line produced. Q E D.

Cor. 1. To construct the Locus of the Point E.
Let BK. CL & CK be respectively y° Axis & ½
Let BK be taken $= Bb \times \frac{t}{m}$ & $CK = Cb \times \frac{t}{m}$. Now
is evident, that the Points B, K will be in
the Circumference of the Circle required —
Dr. Because in y° Line BK. bK.
that Point K. the Points D & E will be in
both the Line Db & the Circumference
of the Circle, & therefore in their Intersection.
Cor. 2. To draw the Perpendicular DF cutting
the Line Bb produced if need be in F. Then
with a Circle upon the Centre F and the Dis-
tance FB or FK be the Circle required —
Cor. 2. The Triangles BFD. bFK. DCb. DFC.
FCB, are all similar to one another. For they have
all one right Angle and the sides about it proport.
Db. bK :: FC. CB. by the Construction. — —
Cor. 3. Joyn. DK & KC. & the Triangle KDC-
will be similar to the Triangle DEC. for
KD. DC :: BE. EC & KD. DC :: DE. EC
whose ratio as equal, for in both KD. DC.

Let the ellipse ABab by a Revolution about its lesser Axis, generate a Spheroid, whose equal Particles attract at equal Distances in the Inverse Duplicate Ratio of their Distances. It is required to determine the Law which a Particle of matter placed any where in the Axis BG produced, will be attracted by a duplicate Spheroid.

Retaining the Construction of the foregoing Prop: from the Point E, in the Axis CE let a new Ordinate edg be drawn infinitely near the former. Dg, & let the Spheroid generated by the Revolution of the infinite Portion of this Curve whose Planes are all perpendicular to the Axis BG, & let the Spheroid generated by the Revolution about the Axis CE of the Figure CEeg be found; which Spheroid will be attracted by the Particle A, in the Direction AE, with a Force proportional to $\frac{AE \times Ee}{DE}$ (for the Attraction of this Spheroid is to the Attraction of the Sphere described with the Radius DE, as the Solid generated by the Figure CEeg is to the Solid generated by the Figure CDdg, by Cor. 1. Prop. 34. Book 1. Princip.: But the Attraction of this Sphere is to the Attraction of the whole Sphere ABab, as $\frac{AE \times Ee}{DE}$ is to $\frac{AB \times Bb}{DB}$ (for the Attraction of the whole Sphere ABab is to the Attraction of the Sphere described with the Radius DE, as $\frac{AB \times Bb}{DB}$ is to $\frac{AE \times Ee}{DE}$) and therefore the Attraction of this Sphere is to the Attraction of the whole Sphere ABab, as $\frac{AE \times Ee}{DE}$ is to $\frac{AB \times Bb}{DB}$. Wherefore the Attraction of the Sphere will be as $\frac{AE \times Ee}{DE}$. Let $\frac{Ae \times Ee}{De}$ represent the Attraction of this Sphere in any other Place e, & the Attraction of the whole Sphere ABab will be found by the Sum of all the Fluxions, or the Area which is to the Area of the Circle described with the Radius AB, as the Area which represents the Attraction of this Sphere is to the Attraction of the whole Sphere ABab. Now, to find the Area which represents the Attraction of this Sphere, let $\frac{AE \times Ee}{DE}$ be the Fluxion of the Area, & the Area will be found by the Fluent of this Fluxion. Let $AE = z$, $DE = x$, & $Ee = \dot{x}$, & the Fluxion of the Area will be $\frac{z\dot{x}}{x}$. Now, to find the Fluent of this Fluxion, let $DE = x$, $DC = 1$, $CE = y$, & $Ee = \dot{y}$, & the Fluxion of the Area will be $\frac{\dot{y}}{x}$. Let $Dd = \dot{x}$, & $dg = \dot{z}$, & $Eg = \dot{v}$. Then will the Triangles EFg & DE be similar, and,

we shall have fg as ef to gh as Ef is to Bb. Now
whose $Bb \times de = Ef \times gh = Df \times gh \times \frac{c}{m}$.
Therefore, $\frac{Bb}{Df} \times de = gh \times \frac{c}{m}$. & the sum
of all the Flexions $\frac{Bb \times de}{Df}$ will = the sum of
all the Flexions $kf \times \frac{c}{m}$ generated in $\frac{1}{2}$
same time. But as must be the latter same as
make for our Purpose, in $bk - Bk \times \frac{c}{m} =$
$Bb - Bb \times \frac{cc}{mm} = Bb \times \frac{cc}{mm}$. Therefore the
2d part of our Flext. is $Bb \times \frac{cc}{mm}$ add this
to Bb the 1st Part already found & they will
make $Bb \times 1 + \frac{cc}{mm} = bb \times \frac{mm+cc}{mm} = bb \times \frac{tt}{mm}$
$= \frac{2ctt}{mm}$. Call $\frac{ctt}{w}$ w. is always proportl to
the magnitude of the Flexion fg. And y
the whole Flext. of the Flemt add $bb =$
gether, will make $\frac{2y}{mm}$.
$\frac{1}{2}$ bc now come to the 3d Part of the
Flemt. w. is to be investigated from the former
& w. is the Flexure of $\frac{Df \times de}{Df}$. And the
Flexure be obtained by the help of the same
similar Principles, fh, g, & Ef where fg or
$de : fg :: Ef : Bf$. Then we $Bf \times de = fg \times Ef$
$= Df \times fg \times \frac{c}{m}$ therefore $\frac{Eg}{Df} = \frac{fg}{Bf} \times \frac{c}{m}$. &
$\frac{Df \times de}{Df} = Df \times \frac{c}{m} \times \frac{fg}{Bf}$ and the Flexure
of the former = the Flexure of the latter.
w. is $Df \times \frac{c}{m} \times \frac{fg}{Bf}$. Suppose Df to be $= k$.
& upon the Centre f with the Radius Df
describe the Circle cutting Df & Bf in d & e
respectively, & because the Angle $Ddk = Bfb$
(by Cor 2d Lemt Sect. 2) the Arcs dg & eb

will be similar & we shall have $\frac{KEL}{GH} = \frac{LEL}{DC}$
& $DH \times \frac{c}{m} \times \frac{KEL}{GH} = \frac{DK}{DC} \times \frac{c}{m} \times LEL$. but $DK : DH ::$

::

wch be similar to magnitude here. $\frac{dv}{dt} = \frac{dv}{dt}$
& $dH \times \frac{c}{m} \times \frac{dv}{dt} = \frac{dv}{dt} \times \frac{c}{m} \times \frac{dv}{dt}$ but
$dv : dv :: t : m$ & $dv : dt :: t : m$. log. $dH : dC ::$
" $t : mm$. & $dH = \frac{tt}{mm}$ log. $\frac{dH}{dC} \times \frac{c}{m} \times \frac{dv}{dt} =$
$= \frac{ctt}{m^3} \times \frac{dv}{dt} = \frac{2g}{m^3} \times CK$. In the same ratio
put a line as CO to the cube = to the line CK
& the 3d part of the Abscissa to the radius
for the two former write $\frac{2g}{m^3} \times CO$. subduct
therefore this from $\frac{2g}{m^3} \times CO$, from the same
cube former & was, $\frac{2g}{mm}$. or $\frac{2g}{m^3} \times CH$ there
will remain, $\frac{2g}{m^3} \times HO$ for the attraction of the
whole sphere. Therefore the attraction of
the sphere will be as its magnitude
& the line HO directly, & as the Cube of the line CH inversly.
i.e. the attraction will be as $\frac{2g}{CH^3} \times HO$ & matter
is supposed as the $\frac{2g}{CH^3}$ to get the
Cor. 1. since CO is = to & the whole Tangent is
CH by a series formerly computed, it follows.
$\frac{1}{2}CO = CH - \frac{CH^3}{3CB^2} + \frac{CH^5}{5CB^4}$ &c. Therefore $\frac{1}{2}CO -$
$\frac{CH^3}{3CB^2} - \frac{CH^5}{5CB^4} = V$. Let CH vanish, then the
term $\frac{CH^5}{5CB^4}$ and all y^t follow it will vanish
and we shall have $\frac{HO}{CH^3} = \frac{1}{3CB^2}$.

Cor. 2. The Attraction of the Sphere will be
to y^e Attraction of y^e same matter condensed into
an infinite y^e male Particle in the Centre

as $\frac{\text{Fo}}{\text{Ch}^3}$ to $\frac{1}{3}$ Ch^2. Thus from the surface going towards the figure of the Cond. Sph. the Ath. is accelerated, & in the same manner as a body projected vertically upwards in vacuo loses the same quantity of its Ath. in equal times.

Cor. 3. The Attraction of every sphere of matter is the same as if all the matter were accumulated into its Centre, & by Cor. 2? foregoing the Attraction of a sphere decreases in the duplicate ratio of the distance from the Centre of the sphere & upon the Qty of matter in the sphere & not upon its extension.

Cor. 4. Since the Qty of matter in the sphere is to the Qty of matter of an homogeneous sphere upon the same dist. BC, as BC^3 to BC^3; & Cor. 3 the Attraction of the former sphere is to the Attraction of the latter as BC^2 to BC^2 it follows that the Attraction of the heterogeneous sphere upon the same dist. is as $\text{BC}^2 \times \frac{\text{Fo}}{\text{Ch}^3}$ to $\frac{\text{Ch}^2}{3\text{Ch}^2}$.

Scholium 1st

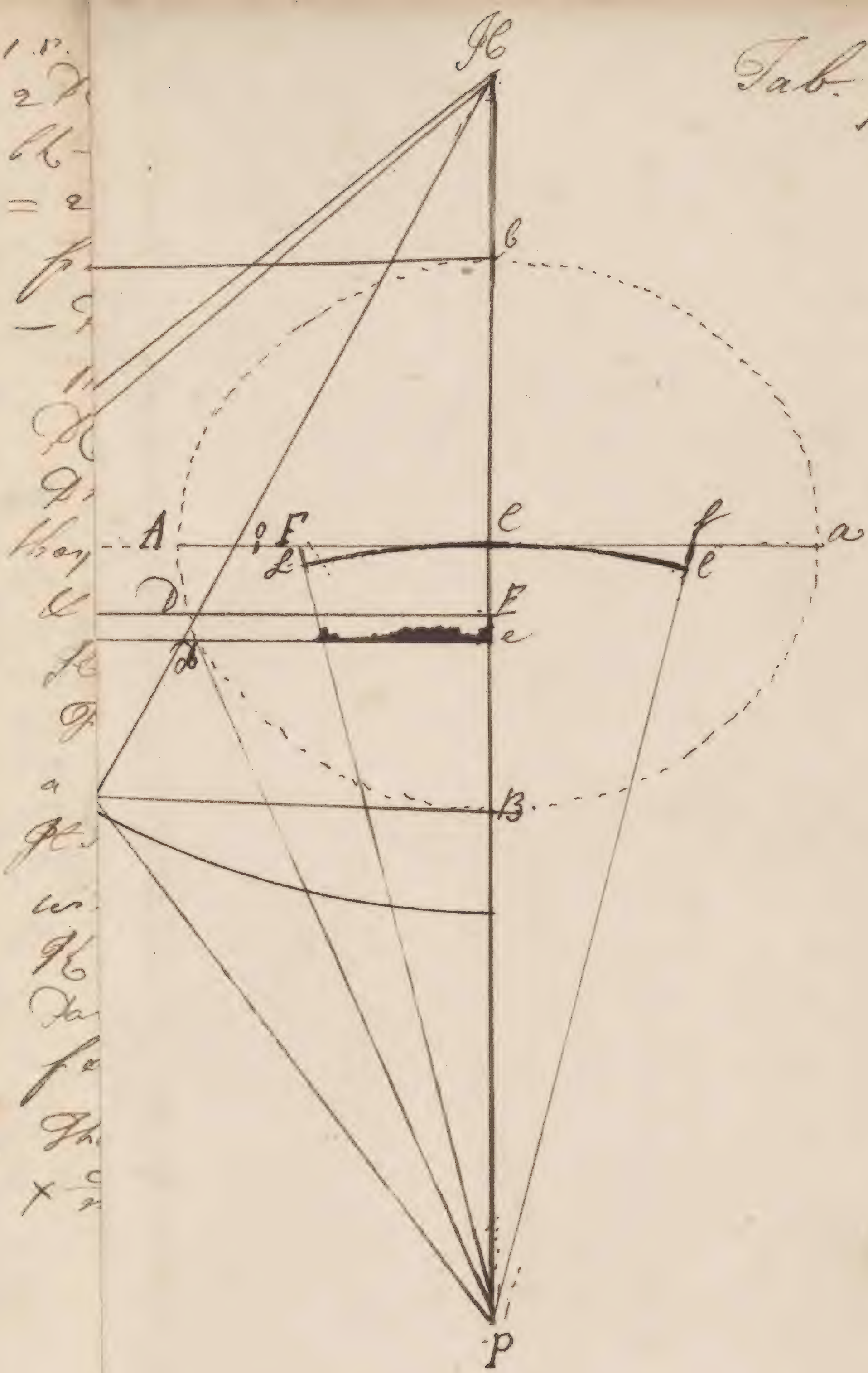
If the 2 last Cor. of this proposition do not carry unequal, & the Prop. of the last Cor. is to be computed, there is no need of having recourse to Trigonometrical Tables.

for the Calculus, may be performed much more expeditiously by the Laws of the 1st Cor. make $AC^2 = p$
$p \times \frac{CF^2}{CD^2} = g$, $g \times \frac{CF^2}{CD^2} =$ &c. Now will the
Attraction of the Spheroid be to the Attraction
of an Homogeneous Sphere upon the same Axis
as $\frac{p}{3} - \frac{q}{5} + \frac{r}{7} - \frac{s}{9}$ &c is to $\frac{BE^2}{3}$ Ex. gr. Let
$BE = 100$ & $AE = 101$, & Cons. $CF = 20\frac{1}{2}$. & let
the Particle P be placed at the vertex B. Then
the whole Case $p = 10201$, $q = 205$ & $r = 4$ & AE^2
$= 10000$ & the Attraction of the Spheroid will be
to the Attraction of the Sphere as $\frac{10201}{3} - \frac{205}{5} + \frac{4}{7}$
: $\frac{10000}{3}$ (2. 10201 — 123 + 2 : 10000 (2. 10080:
10000, or as 126. 125 —

Scholium 2d —

Let the Attracted Particle P be placed
within the Spheroid, suppose between B & C, & let
BK & GK be taken as before both ways same
side of Axis, & the Lines PK and PL make
now the Diff. Lines, measuring segm.
whose Area is PK the Fluxion $Ee - \frac{PK \times Ee}{PD} =$
$Ee + \frac{PK \times Ee}{PD} - \frac{PH \times Ee}{PD}$ as before.
But in the other segm. whose Area is PB
the Fluxion $Ee \frac{PB}{PD} Ee = Ee - \frac{PK \times Ee}{PD} +$
$+ \frac{PH \times Ee}{PD}$ the same drawing PH perpen.
to BK as for as the Centr. K. Then
Attraction of the segm. whose Axis is PB, will
be $= PG \times BK - PG \times \frac{c}{m} - PH \times \frac{c}{m} \times \frac{PK}{PK}$
or $+$
and the Attraction of the other segm. whose
Axis is PL will be $= PB \times BK - PG \times \frac{c}{m} \times \frac{PK}{PK}$
or $+$
subtract the 2d Attraction from the former

Tab. 9. Page 44.

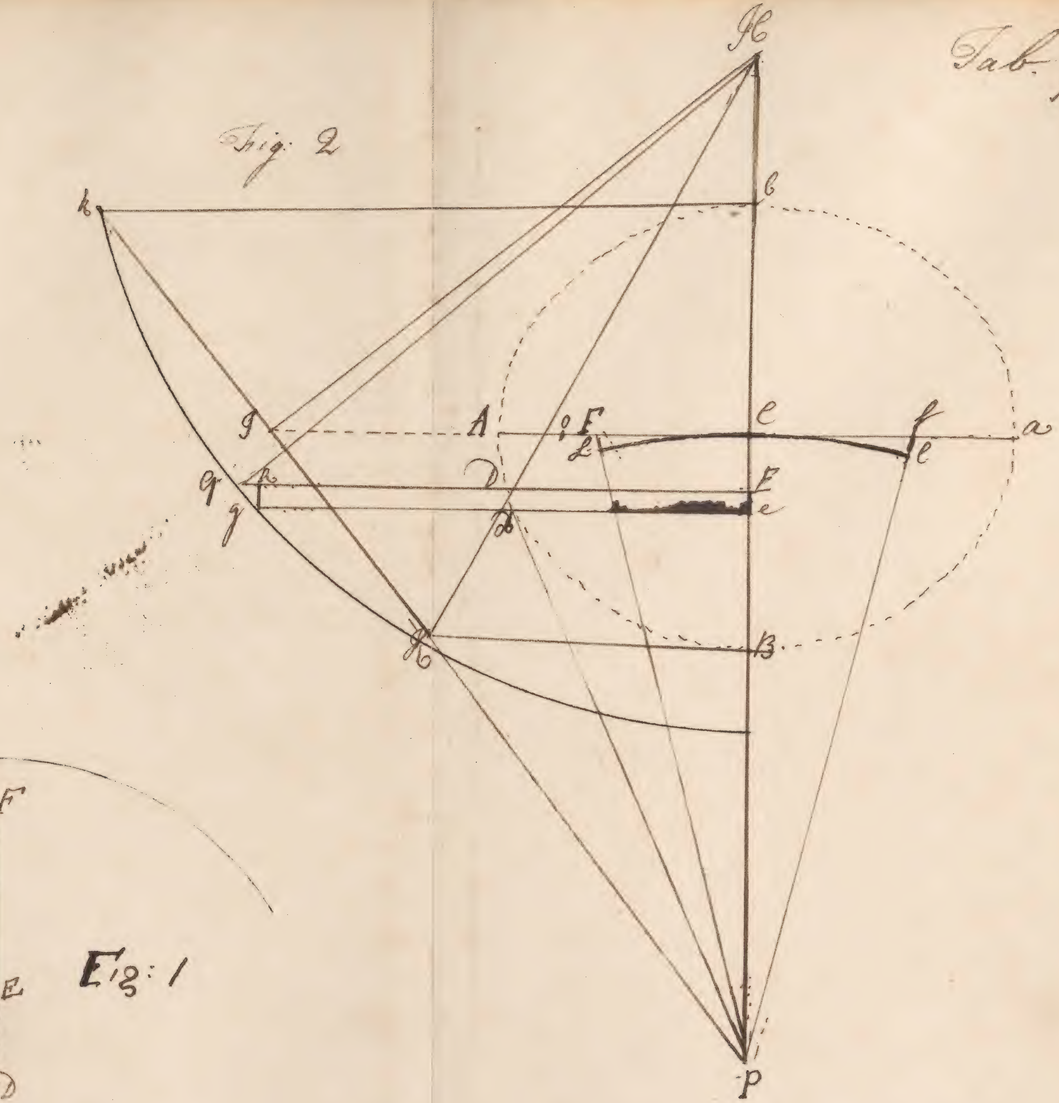


1. P. subtract Db from Hb & there will remain
2 HC Again subtract $Bk - Bb \times \frac{c}{m}$ from
$kk - bb \times \frac{c}{m}$, & there will remain $kk - Bk \times \frac{c}{m}$
$= 2HC \times \frac{cc}{mm}$. Lastly subtract $Hh \times \frac{c}{m} \times \frac{2Hk}{Hk}$
from $- Hh \times \frac{c}{m} \times \frac{Bk}{Hk}$. & there will remain
$- Hk \times \frac{c}{m} \times \frac{Hk}{Hk}$.

Now that this last part is as the distance HC as well the 2 former & the Demonstration Produce the Chord Hk & the Ellipsis Bbk till they meet in 2. & Draw kl & bm parallel to Q2. as Cp. Cq. & Q2: pk :: Q2: Cb therefore as $\frac{cc}{mm}$: $\frac{Hk}{Hh}$ :: Cp: Cb. but Cb is a constant Q.E. for Cp or $\frac{Bk + Ck}{2}$ or $\frac{1}{2} Bk \times \frac{c}{m}$ is a constant Q.E. therefore the Triangle HBk, & consequently the Triangle Hkk will be equal in Area, & the Angle HHk will be the same wherever the Particle P. is placed between B & b. therefore $\frac{Hk \cdot Hk}{Hh}$ will be a constant Q.E. Therefore $Hh \times \frac{c}{m} \times \frac{Hk}{Hk}$, or $2HC \times \frac{cc}{mm} \times \frac{c}{m} \times \frac{Hk}{Hk}$ will be as HC

Tab. 9. Page 144

Fig: 2



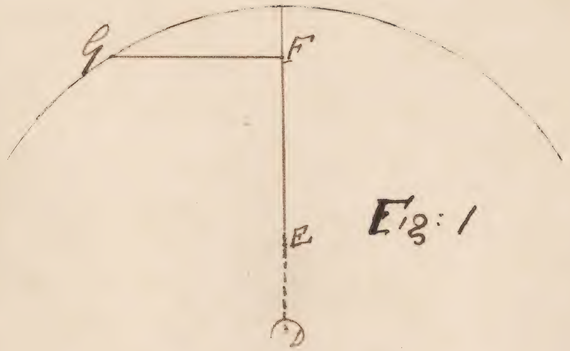


Fig: 1

NB. By the Determination of a force is to be understood the Line which it acts in, or any other Line || to it. Thus if a force acts from A towards B, & any Line as EF be drawn parallel to AB, it may be valued the same way w.th Respect to EF as AB is with Respect to A, for they may the Line EF be made to denote the Determination of a force acting from A. & a contrary Determination.

Prop.n 1. Fig.r 1. 2. Tab.r 1. 2.d Page 21

If there be a Triangle EFG, whose 2 Sides EF & FG represent both the Direction & Determination of 2 forces, acting from a given Point as A. & if the 3.d Side EG, wch represent both the Direction & Determination of a 3.d force, wch acting from the same Point A, will be equivalent to the other two. For drawing AB = & || to EF. & AC = & || to FG. & completing the Parallelogram ABDC. The Lines AB & AC will represent both the Direction of the forces, acting from A. & two forces will both together be equivalent to the single force AD, as is evident from the Construction of Newt. Law. respectively, the Angle EFG will = ∠ ABD, therefore the Side EG will = the Side AD, & ∠ FEG = BAD. Therefore the Lines EF & FG have the same Determination w.th the Lines AB & BD, the Line EG will be also parallel to AD. Therefore the Line EG will represent both the Direction & Determination of the force AD.

Cor.l 1. Fig.r 3 Tab.l 1. Page 21

In the manner it is evident, that as many forces acting from the same Point A, & of wch the Directions & Determinations be represented, from so many contiguous Sides, EF, FG, GH, HI, & the Line EI be drawn from the first Point E to the last I.

Lemma represent. both the 2 Pts & Determination of a simple 146
force, w. acting for A. will be equivalent to all the forces
put together. For the forces, 2 & 3 are equi: to 4, by
1st Prop. & for 4th same Reason. the forces 2 & 5 & 6, are equi: to
4th force. & the forces 2 & 6 & 7 to the 5th force &c. & the
forces, 6 & 7, 7 & 8, 8 & 9, are equivalent to 4th force &
Cor 2. If 4th last Point remains w.t A, the Prop.
& 4th forces acting at A, will have no Effect, but will
sustain each other in Equilibrio —
Cor 3. On the other Hand, if 2 & 3 & 8 & 9 be a Polygon
whereof any one side, as 8 & 9 represents both the Pts & Determination of a simple force acting from Point A,
& other sides, 2 & 3, 3 & 4 &c. will represent & Pts & Determinations of so many other forces, w. all acting from the same point
A, will Mag: be equi: to the former simple one —

Of the Resist of Heavy Bodies
Prop: 5 —

The Acc. generated in any Pt of matter, by any
constant action of an uniform force or Pt continued for
the time t, will be as the Pt, for it must be (rat: hor:)
as Velocity, as t Velocity & as Mag: conjointly. —
Cor 1. If 4th moving force, & be as the Pt of matter
the Pt will be constant, in w.t Case the Acc. will
be as 4th time t. & in 4th same time will be always
the same —
Cor 2. Therefore if any nos of Bodies, how diff. soever, in
their Kind, and Pts of matter, be acted upon by forces prop: to
their matter, they will be equally accelerated & Comp: hyp:
this equal Spaces, in equal times —
Cor 3. If any nos of Bodies how heterogeneous soever,
they may be so one another, being acted upon by diff. forces
to found to be equally accelerated, i.e. hyp: this: = spaces, i = times
in a prop: ratio nos forces w.t w.h these Bodies are moved
are prop: to the Pts of matter they contain —

Scholium

The Qty v is usually called 4° vis motrix & the Qty $\frac{v}{t}$ 4° vis acceleratrix because 4° former is always prop. to the motion generated in a given time, the latter to the Vel. it has 4° specific Gravity of a Body is not its gravity absolutely, but its gravity w.t respect to its magnitude, so 4° vis acceleratrix of a Body is not the vis motrix absolutely but the vis motrix with respect to the matter. It is 4° sense of the word our Philosophic vis Insita, thus, viz. The Vel. generated by the constant Action of an uniform force, continued for any time, wch is as the product of the time × the accelerating force, & therefore in Cases wh.e the accel: force is the same the vel: generated wil be as 4° Time.

Prop. 3

The weight of all Bodies is prop.l to 4° Quan. of matter. This appears from equal acceleration of Bodies (in vacuo) of all kinds, & is further confirmed by experiments made upon the Oscillations of Pendulums.

Supp.e Huygens has demonstrated yt as the Qty of the Gravity of any Circle is to the Qty of its Circumference so is half the Length of any Pendulum, to the Space thro' wch a heavy Body falls, yt is at rest, by & made to fall, by 4° same weight wch before produced its Oscillations, provided the time of the fall be equal to the time of one Oscillation of 4° Pendulum. Thus if D be the Diam.r & C the Circumference of any Circle, & L the Length of any Pendulum, & T the time of one Oscillation, the Space thro' wch a heavy Body falling by its weight in this time T, will be $\frac{1}{2} L \times \frac{CC}{DD}$ so yt knowing the Quantities how far wil a heavy Body fall in a sec.d of time, is now reduced to this wch is the Length of a Pendulum swinging seconds. Now if the Length of a Pendulum swinging seconds be found to be 39, 2 Inches & the Diam.r & Circumference of a Circle 1. 22. the Space thro' wch a heavy

Body defends in any reg. of time wth the Body found by the foregoing Proportion to be 16. 1. 6. 148
Prop. 4.

The gravity between the Earth & its parts is mutually equal.

To demonstrate this let the Globe of the Earth be conceived to be divided into 2 Parts equall or unequall, now if either of those segments should gravitate more upon the other, than the other upon it, there would be a greater Pressure one way than another, & the Earth giving way to the greater pressure, would fall down & its place in infinitum. Not to say that upon this supposition of a Body at rest w.ch has also to put itself in motion w.out the action of any external power whatsoever, w.ch is absurd. — Vide Coles Defan. to Newt. Princip. & Newt. gen. sch. to Laws of Nature.

Prop. 5.

The Forces wherewith Bodies attract the Earth, are prop.l to their Quantities

For the Forces wherewith Bodies attract y.e Earth, are equal to the forces, wherewith, the Earth attracts y.m. (by the last) but those latter forces, are prop.l to the Qty of matter in the Bodies attracted (by y.e Prop.) therefore the former are to be as the Qties of Matter wch Cor. 1. as the forces wherewith every Body attracts the Earth, or is attracted by it is owing (not to its matter as such, and not to any particular constitution of the Earth, or any magnetic Body in its Centre, but to the mass of matter whose Centre is Cor. 2. since the Attraction of the whole Earth, arises from the Attraction of all its parts, and since all Bodies, as much those of the Body upon its surface, as those of it adhere to it,

may be look'd upon as parts of the whole
Earth, it follows y^t the Bodies wherof an
Earth is compos'd, mutually & equally attract
one another, tho' y^t Attraction is too small to
have any sensible effect

Prop: 6.

If any Heavy Body as g, press'd by its own
weight, descends thro' a Space s, in y^e time t,
& at last acquires the Vel: V, I say then y^t
the Space s, is but ½ the Space it wou'd
have made, had it mov'd all along ab the time
wth the Vel: v.

For since the Vel: of a falling Body increases
uniformly, wth the time of the fall, & since
between the first Velocity which was *